STUDENT SUPPORT MATERIAL

CLASS XII

MATHEMATICS

Session: 2020-21

KENDRIYA VIDYALAYA SANGATHAN

REGIONAL OFFICE LUCKNOW

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Mathematics

Class XII

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MATHEMATICS SYLLABUS (REVISED)

CLASS-XII
(2020-21)

One Paper **Max Marks:80**

No.	Units	No. of Periods	Marks
I.	Relations and Functions	17	08
II.	Algebra	35	10
III.	Calculus	57	35
IV.	Vectors and Three - Dimensional Geometry	26	14
V.	Linear Programming	13	05
VI.	Probability	20	08
	Total	168	80
	Internal Assessment		20

Unit-I: Relations and Functions

1. Relations and Functions **9 Periods**

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions.

2. Inverse Trigonometric Functions **8 Periods**

Definition, range, domain, principal value branch.

Unit-II: Algebra

1. Matrices **17 Periods**

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices, Invertible matrices; (Here all matrices will have real entries).

2. Determinants **18 Periods**

Determinant of a square matrix (up to 3 x 3 matrices), minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

1. Continuity and Differentiability **16 Periods**

Continuity and differentiability, derivative of composite functions, chain rule, derivative of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives.

2. Applications of Derivatives **7 Periods**

Applications of derivatives: increasing/decreasing functions, tangents and normals, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. Integrals **15 Periods**

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^{2+bx+c}}}$$

$$\int \frac{px + q}{ax^2 + bx + c} dx, \int \frac{px + q}{\sqrt{ax^{2+}bx + c}} dx, \int \sqrt{a^2 \pm x^2}\, dx, \quad \int \sqrt{x^2 - a^2} dx$$

Fundamental Theorem of Calculus (without proof).Basic properties of definite integrals and evaluation of definite integrals.

4. Applications of the Integrals **9 Periods**

Applications in finding the area under simple curves, especially lines, parabolas; area of circles /ellipses (in standard form only) (the region should be clearly identifiable).

5. Differential Equations **10 Periods**

Definition, order and degree, general and particular solutions of a differential equation. Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree of the type: $\frac{dy}{dx} = f(y/x)$. Solutions of linear differential equation of the type:

$$\frac{dy}{dx} + py = q, \text{ where p and q are functions of x or constant.}$$

Unit-IV: Vectors and Three-Dimensional Geometry

1. Vectors **13 Periods**

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

2. Three - dimensional Geometry **13 Periods**

Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Distance of a point from a plane.

Unit-V: Linear Programming

1. Linear Programming **13 Periods**

Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems. graphical method of solution for problems in two variables, feasible and infeasible regions (bounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

1. Probability **20 Periods**

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution.

Prescribed Books:

1) Mathematics Textbook for Class XI, NCERT Publications
2) Mathematics Part I - Textbook for Class XII, NCERT Publication
3) Mathematics Part II - Textbook for Class XII, NCERT Publication
4) Mathematics Exemplar Problem for Class XI, Published by NCERT
5) Mathematics Exemplar Problem for Class XII, Published by NCERT
6) Mathematics Lab Manual class XI, published by NCERT
7) Mathematics Lab Manual class XII, published by NCERT

MATHEMATICS (Code No. - 041)
QUESTION PAPER DESIGN CLASS - XII
(2020 - 21)

Time: 3 hours

Max. Marks: 80

S. No.	Typology of Questions	Total Marks	% Weightage
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	44	55
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	20	25
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	16	20
	Total	80	100

1. *No chapter wise weightage. Care to be taken to cover all the chapters*
2. *Suitable internal variations may be made for generating various templates keeping the overall weightage to different form of questions and typology of questions same.*

Choice(s):

There will be no overall choice in the question paper.

However, 33% internal choices will be given in all the sections

INTERNAL ASSESSMENT	**20 MARKS**
Periodic Tests (Best 2 out of 3 tests conducted)	10 Marks
Mathematics Activities 10 Marks	

Note: For activities NCERT Lab Manual may be referred

DELETED TOPICS FOR SESSION 2020-21

UNIT/CHAPTER	SYLLABUS REDUCED
Unit I: Relations and Functions	
1. Relations and Functions	• composite functions, inverse of a function.
2. Inverse Trigonometric Functions	• Graphs of inverse trigonometric functions • Elementary properties of inverse trigonometric functions
Unit II: Algebra	
1. Matrices	• existence of non-zero matrices whose product is the zero matrix. • Concept of elementary row and column operations. • proof of the uniqueness of inverse, if it exists.
2. Determinants	• properties of determinants • Consistency, inconsistency and number of solutions of system of linear equations by examples,
Unit-III: Calculus	
1. Continuity and Differentiability	• Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretation.
2. Applications of Derivatives	• rate of change of bodies, use of derivatives in approximation
3. Integrals	• $\int \sqrt{ax^2+bx+c}\,dx$ • $\int (p\,x+q)\sqrt{ax^2+bx+c}\,dx$ • Definite integrals as a limit of a sum
4. Applications of the Integrals	• Area between any of the two above said curves
5. Differential Equations	• formation of differential equation whose general solution is given. • Solutions of linear differential equation of the type: $\frac{dx}{dy}$ +Px=Q, where P and Q are functions of y or constants.
Unit-IV: Vectors and Three- Dimensional Geometry	
1. Vectors	scalar triple product of vectors.
2. Three - dimensional Geometry	• Angle between (i) two lines, (ii) two planes, (iii) a line and a plane
Unit-V: Linear Programming	
1. Linear Programming	• mathematical formulation of L.P. problems • (unbounded)
Unit-VI: Probability	
1. Probability	• mean and variance of random variable. Binomial probability distribution.

STRATEGY FOR COMMON ERRORS/ COMMITTED MISTAKES AND THEIR REMEDIES

Chapter - 1
RELATIONS AND FUNCTIONS

Common Errors

1. Students unable to understand difference between Reflexive, Symmetric, and Transitive relation.
2. They do not give Proper Reasoning for Reflexive, Symmetric, and Transitive relation.
3. In composition of function students committed mistakes on applying order of function.
4. They committed mistake to prove a function as onto.
5 . They can't differentiate between relations and Functions.

6. Finding of domain and range.

Remedy

1. Students should practice and differentiate Reflexive, Symmetric, and Transitive relation.
2. Students learn how to give Proper Reasoning for questions based on Reflexive, Symmetric, and Transitive relation
3. In composition of function like gof (x) , first apply f rule and then g rule on x.
4. Focus on how to Prove a function as Onto.
5. To clear concepts of domain and range.

SOME EXAMPLES:

1. Let A = { 1,2,3}. Check whether R = { (1,2),(2,1), (1,1), (1,3)} is symmetric or not.
Mistake done : Here, (1,2)$\in R$, (2,1))$\in R$. So, it is symmetric.
Correction : The student thinks that only an ordered pair is to check for Symmetric. So, he forgets to check for (1,3).

2. If A = {1,2,3}, check whether R = {(1,1),(1,2),(2,1)} is transitive or not.
Mistake done: (1,2))$\in R$, (2,1))$\in R$ implies that (1,1))$\in R$. So it is transitive.
Correction: Here the student forgets to see for (2,1))$\in R$, (1,2))$\in R$ implies (2,2) $\in R$or not.

3.Consider f : $R_+ \to [-5,\infty)$ given by f(x) = 9x^2+6x-5. Show that f is invertible..
Solution: Let y be an arbitrary element of $[-5 , \infty)$

and y = 9x^2+6x-5

$\therefore x = \left(\frac{(\sqrt{y+6})-1}{3}\right)$

Now, let us define a function g: $[-5 , \infty) \to R_+$

Such that g(y) = $\left(\frac{(\sqrt{y+6})-1}{3}\right)$

Then gof(x) = gf(x) = g(9x^2 + 6x – 5)
= g((3x +1)^2 – 6) =x

{ This step is very important. Most of the students make mistake at this step. Students must have to be very careful while computing gof(x)}

And fog(y) = f(g (y)) = f $\left(\frac{(\sqrt{y+6})-1}{3}\right)$ =y

fog = gof = I_{R+}

Hence f is invertible.

4.Show that relation R in the set A =$\{1,2,3,4,5\}$ given by R = $\{(a, b): |a - b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of $\{1,3,5\}$ are related to each other and all the elements of $\{2,4\}$ are related to each other. But no element of $\{1,3,5\}$ is related to any element of $\{2,4\}$.

Solution: For every element a $\in A$, We have $|a-a|=0$,
Which is even.

$\therefore R$ is reflexive.

$|a-b|$ is even

$|b-a|$ is even

$(b,a) \in R$

$\therefore R$ is symmetric.

$|a-b|$ is even and $|b-c|$ is even

$(a-b)$ is even and $(b-c)$ is even

$(a-b)+(b-c)$ is even

$a-c$ is even.

$|a-c| \in R$

$\therefore R$ Is transitive

Hence, R is equivalence relation.

Now, all elements of the set $[1,3,5]$ are related to each other as all the elements of this subset are odd Thus, the modulus of the difference between any two elements will be even. Similarly, all elements of the set $[2,4]$ are related to each other as all the elements of this subset are even.

Also no element of the subset $[1,3,5]$ can be related to any element of $[2,4]$ as all the elements of $[1,3,5]$ are odd and elements of $[2,4]$ are even.Thus the modulus of the difference between the two elements one from each of these two subset will not be even.

Generally, students didn't understand the concept of Equivalence Class. The concept should be cleared, so that they can get full marks.

Chapter - 2
INVERSE TRIGONOMETRIC FUNCTIONS

1. **Error**: $sin^{-1}x = \frac{1}{\sin x}$ or $sin^{-1}x = \left(\frac{1}{sin}\right)x$

Remedy: Students must go through the concept of invertible functions given in Chapter 1 (Relations and Functions) of class 12.

If $f: X \to Y$ and $g: Y \to X$ are two functions such that $fog = I_Y$ and $gof = I_X$ then f is said to be invertible and g is the inverse of f; written as $g = f^{-1}$, (f^{-1} is the symbolic representation for inverse of f. It is not "-1 raised to the power of f")

$\therefore fof^{-1} = I_Y$ and $f^{-1}of = I_X$

Similarly, sin^{-1} function is defined as the inverse of sin function.

2 **Error**: $cos^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{3}$

Remedy: $cos^{-1}\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Students must know principal value branch of inverse trigonometric functions

Function	Principal Value Branch
$sin^{-1}x$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$cos^{-1}x$	$[0, \pi]$
$tan^{-1}x$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$cot^{-1}x$	$(0, \pi)$
$sec^{-1}x$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$cosec^{-1}x$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

3. **Error**: $sin^{-1}\left(sin\frac{3\pi}{4}\right) = \frac{3\pi}{4}$

Remedy: $sin^{-1}\left(sin\frac{3\pi}{4}\right) = \frac{\pi}{4}$

Students must know following rules

$sin^{-1}(\sin x) = x \text{ if } x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $sin(sin^{-1}x) = x \text{ if } x \in [-1, 1]$

$cos^{-1}(\cos x) = x \text{ if } x \in [0, \pi]$, $cos(cos^{-1}x) = x \text{ if } x \in [-1, 1]$

$tan^{-1}(\tan x) = x \text{ if } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $tan(tan^{-1}x) = x \text{ if } x \in \boldsymbol{R}$

4. Students must learn domain and range of Inverse Trigonometric Functions

Function	Domain	Range (Principal Value Branch)
$sin^{-1}x$	[-1, 1]	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$cos^{-1}x$	[-1, 1]	$[0, \pi]$
$tan^{-1}x$	**R**	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$cot^{-1}x$	**R**	$(0, \pi)$
$sec^{-1}x$	**R** – (-1, 1)	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$cosec^{-1}x$	**R** – (-1, 1)	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Chapter - 3
MATRICES

Concept	Probable Errors	Action taken by teachers	Important Problems of NCERT Mathematics Text Book
Order of matrix Equality of matrices	Taken row as column and column as row Without seeing order, equating elements	Make them understand row means horizontal, column means vertical more such problems for practice. First see same order, then equate corresponding elements	**Ex 3.1** **Q. No 4,6,7,9,10**
Addition of Matrices Product of Matrices	Without seeing order, add elements Not multiplying the first matrix row elements with the second matrix corresponding column elements and add	First see same order, then add corresponding elements By giving tips like Run and Jump remember while multiplying two matrices. More practice on various order matrices for multiplication.	**Ex 3.2** **Q. No 6,7,9,13,16,17, 18,20,21,22**
Transpose of a Matrix	Converting both column into rows and rows into columns	Stress to be given only to change row into columns or vice versa but not both	**Ex 3.3** **Q. No 7,8,9,10,11,12**
Adjoint of a Matrix	(a) Students find co-factors without taking proper sign (b) Students don't take the transpose of a matrix made by the co-factors.	Teachers tell students to find co-factors by using proper sign and insist them to take transpose of a matrix made by the co-factors by giving more questions for practice	

Chapter - 4
DETERMINANTS

1. Students don't apply row and column operations properly when they are evaluating the determinants.
2. Students miss writing the formula when they are solving the system of equations by matrix method
3. Students commit mistakes in inserting the sign when they are finding the co-factors.
4. Students do not apply property of determinants properly.

Concept/Question:	Some students write it as:	Correct solution / statement is:
1. Give an example of determinant of order 3.	$A = \begin{bmatrix}3 & 4 & 2\\ 1 & 0 & 6\\ -1 & 1 & 2\end{bmatrix}$ It is wrong representation for a determinant. It is representation of matrix.	Correct representation is $\lvert A\rvert = \begin{vmatrix}3 & 4 & 2\\ 1 & 0 & 6\\ -1 & 1 & 2\end{vmatrix}$
2. Evaluate $\begin{vmatrix}2 & -1 & 4\\ 0 & 1 & 6\\ 1 & -3 & 2\end{vmatrix}$	$\begin{vmatrix}2 & -1 & 4\\ 0 & 1 & 6\\ 1 & -3 & 2\end{vmatrix}$ = 2x(2+18) -1x (0-6) + 4 x(0-1) = 42 is wrong answer.	For expending a determinant we must assign sign as $\begin{vmatrix}+ & - & +\\ - & + & -\\ + & - & +\end{vmatrix}$ Correct solution = 2(2 + 18) + 1(0-6) + 4(0 -1)=30
3. Evaluate $\begin{vmatrix}1 & 5 & 2\\ 2 & 1 & 7\\ -1 & 8 & 4\end{vmatrix}$	Perform $R_2 \rightarrow 3R_2$ (Multiply element of second row by 3) Δ = $\begin{vmatrix}1 & 5 & 2\\ 6 & 3 & 21\\ -1 & 8 & 8\end{vmatrix}$ is wrong.	Δ =$\frac{1}{3}\begin{vmatrix}1 & 5 & 2\\ 6 & 3 & 21\\ -1 & 8 & 8\end{vmatrix}$ is correct working.
4.Find area of triangle with vertices (4, 1), (4, -2), (0, 6)	Area of triangle = $\frac{1}{2}\begin{vmatrix}4 & 1 & 1\\ 4 & -2 & 1\\ 0 & 6 & 1\end{vmatrix}$ = $\frac{1}{2}$ [4(-2-6)-1(4-0)+1(20-0)] = -8 Area of triangle = -8 is wrong.	Area of triangle = $\frac{1}{2}\begin{vmatrix}4 & 1 & 1\\ 4 & -2 & 1\\ 0 & 6 & 1\end{vmatrix}$ = $\frac{1}{2}$ [4(-2-6)-1(4-0)+1(20-0)] = -8 Area of triangle is = 8 square unit is correct
5. Find the value of k, such that area of triangle with vertices (3, k), (2, 2), (-4, 1) is 3 square units.	Δ= $\frac{1}{2}\begin{vmatrix}3 & k & 1\\ 2 & 2 & 1\\ -4 & 1 & 1\end{vmatrix}$= 3 $\frac{1}{2}$[3(2-1) –k(2+4) +1(2+8)] =3 Or $\frac{1}{2}$[3 –k + 10] =3 13 -6k =6 6k =7 K =$\frac{7}{6}$	Here area of triangle = 3 square units. So values of determinant can be 3 or -3 both. So we must take both case. i.e. $\frac{1}{2}$[3 –k + 10] =3 or 13 -6k =6 or 6k =7 K =$\frac{7}{6}$ and $\frac{1}{2}$[3 –k + 10] =-3 Or 13 -6k =-6 Or 6k =19 K =$\frac{19}{6}$ so k =$\{\frac{7}{6}, \frac{19}{6}\}$
6. Find adjoint of matrix A, where	A_{11}= 5 , A_{12} = -2 A_{13} = -3 A_{21} = 5, A_{22} = -2 , A_{23} = -3 A_{31} = -10, , A_{32} =4,, A_{33} = 6	A_{11}= 5 , A_{12} = 2 A_{13} = -3 A_{21} =- 5, A_{22} = -2 , A_{23} = 3 A_{31} = -10, , A_{32} =-4,, A_{33} = 6

A = $\begin{bmatrix} 2 & 1 & 4 \\ 0 & 3 & 2 \\ 1 & -1 & 1 \end{bmatrix}$	So adj.A = $\begin{bmatrix} 5 & -2 & -3 \\ 5 & -2 & -3 \\ -10 & 4 & 6 \end{bmatrix}$ is wrong.	So matrix formed by cofactor is $\begin{bmatrix} 5 & 2 & -3 \\ -5 & -2 & 3 \\ -10 & 4 & 6 \end{bmatrix}$ after taking transpose of above matrix to get adj.A i.e. $\begin{bmatrix} 5 & -5 & -10 \\ 2 & -2 & -4 \\ -3 & 3 & 6 \end{bmatrix}$
7.Given matrix A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, prove that $A^2 - 5A + 7I = 0$ and hence find A^{-1}	Some students apply $A^{-1} = \frac{adj.A}{\lvert A \rvert}$ In the question finding A^{-1} will be wrong working.	First we will prove $A^2 - 5A + 7I = 0$ $A^{-1}(A^2 - 5A + 7I) = A^{-1}.0$ $(A^{-1}A)A - 5A^{-1}A + 7A^{-1}I = 0$ Or $IA - 5I + 7A^{-1} = 0$ Or $A^{-1} = \frac{1}{7}(5I - A]$ Now we substitute A and I to get A^{-1}.
8. Given A=$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B=$\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & -5 \end{bmatrix}$ Find the product of matrices AB and hence solve the system of equations X - y = 3 2x +3y + 4z = 17 y +2z =7	In this type of problem some of students opts following procedure: Step -1 Find AB Step -2 Find A^{-1} by applying formula $A^{-1} = \frac{adj.A}{\lvert A \rvert}$ Step -3 Find the value of x ,y and z {Most of the students finds correct answer through the above procedure but we have not done according to the question}	Correct procedure is: Step-1 Find AB AB = $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ Or AB = 6 I Step -2 Find A^{-1} AB = 6 I Pre multiply by A^{-1} on both side $A^{-1}(AB) = A^{-1}(6I)$ $(A^{-1}A)B = 6A^{-1}$ $IB = 6A^{-1}$ $A^{-1} = -\frac{1}{6}B$ $=\frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ Step -3 we use the value of A^{-1} in $X = A^{-1}C$ find the value of variables. $X = A^{-1}C$ $= =\frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $=\frac{1}{6}\begin{bmatrix} 2.3 + 2.17 - 4.7 \\ -4.3 + 2.17 - 4.7 \\ 2.3 - 1.17 + 5.7 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$ Or $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ X =2, y =-1, z =4

Chapter -5
CONTINUITY AND DIFFERENTIABILITY

S. No.	Mistakes	Suggestions how to correct the mistakes
1	Sign Error While Solving Modulus functions : While solving the question, the most common mistake done by students is defining the modulus function incorrectly. i.e., when $x < 0$ then treat that the function will give output x, which is incorrect.	$f(x) = \|x\|$ Wrong way: Let x = - 5 So, f(x) = -5 But |x|=|-5|=5 Right way: Let x=-5 So, f(x)= - (- x) = 5 And also |x|=|-5|=5
2	Derivative Error: Students perform some arithmetic operations with the derivative of the function, which is meaningless. Some of those meaningless operations are listed below: $(fg)'(x) = f'(x)g'(x)$ $(f \pm g)'(x) = f'(x) \pm g'(x)$ $(f/g)'(x) = f'(x) / g'(x)$	$(fg)'(x) \neq f'(x)g'(x)$ $(f \pm g)'(x) \neq f'(x) \pm g'(x)$ $(f/g)'(x) \neq f'(x) / g'(x)$
3	Using Chain Rule Incorrectly: $f(x) = \sin\ (\log(x))$ $\frac{d}{dx}(\sin(\log(x))) = \cos\ (\log(x))$ $\frac{d}{dx}(\log(\sin(x))) = \frac{1}{\sin(x)}$ Both the derivative of the functions are incorrect.	$f(x) = \sin((\log(x))$ $\frac{d}{dx}(\sin(\log(x)))$ $\frac{d(\sin(\log(x)))}{d(\log(x))} * \frac{d(\log(x))}{dx}$ $\cos(\log(x)) * \frac{1}{x} = \frac{\cos(\log(x))}{x}$ Similarly, we can find the derivative of second, which comes out to be cot(x).
4	Solving Implicit Function when y is a explicit function of x: Mistake made by students while they are differentiating the above type of function is that they by mistake in rush or unknowingly forget to introduce the $\frac{dy}{dx}$ term when they differentiate both sides w.r.t x. For example: $y^5 = x^2$ Differentiating both sides w.r.t x, $5y^4 = 2x$	$y^5 = x^2$ Differentiating both sides w.r.t x, $5y^4\frac{dy}{dx} = 2x$ we used chain rule here. First, we differentiate y^5 w.r.t to y and then we further differentiated y w.r.t x. That's the reason we have to introduce $\frac{dy}{dx}$ term in the equation.
5	Finding 2nd Derivative When Parametric Function is Given : Let x and y be a function of t. Let $\underline{x = f(t)}$ $\underline{y = g(t)}$ $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ Now when it's the turn to find the 2nd derivative then the most common mistake is,	To solve this problem, we will have to differentiate $\frac{dy}{dx}$. After that we will get and extra term which will be containing dt which can be removed by substitution

	$$\frac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}} = \frac{d^2y}{d^2x}$$ But this is wrong	
6	*Students generally differentiate the constant symbols wrongly.* For example, π = 3.14 But what they do when they Differentiate $\frac{\pi}{4}$ is that they see the symbol of pi and they treat it as a variable and they generally write the answer 1/4.	Differentiation of Constant symbols are same done as constants i.e.,$\frac{d(\frac{\pi}{4})}{dx}$ = 0
7	*Improper use of log during logarithmic Differentiations:* Students generally use wrong log properties For example, $$\log(a+b) = \log a + \log b$$ This is not a property. This is a misconception made by the students	Students should know the correct formula for logarithmic differentiation. Correct formula is: $$\log(a\,b) = \log a + \log b$$
8	In implicit functions, starting the differentiation with dy/dx	Insist them to differentiate the equation directly w.r.t.x
9	In logarithmic differentiation, if y = u+v form, log is taken in the beginning They take logy= logu + logv which is wrong	Teaching basics and insisting to take function in the form y=u + v , u=sinx^x and v=x^x
10	Writing (logx)^n as nlogx	Give the power rule logx^n = n logx
11	For second order derivative in parametric form using d^{2}y/dx^2 is equal to$\frac{d}{dx}(\frac{dy}{dx})$	Insist them to write the formula correctly
12	In Implicit function, starting the differentiation with $\frac{dy}{dx}$	Insist them to differentiate the equation directly w.r.t.x
13	while differentiating inverse trigonometric functions, applying chain rule	More practice with inverse trigonometric functions and chain rule problems
14	For Second order derivative in parametric form – using $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$	Insist them to write the formula correctly

While teaching the chapter Continuity and Differentiability the following common errors/common mistakes done by students, have been observed and the practical remedies is suggested that may somehow minimize the mistakes of students.

1. When the function is defined with different definition in different domain, in that case , the child often get confused to decide which of the part of the function is taken for calculation of left and right hand limit. To minimize this type of mistake, we have to illustrate the concept of limit taking the graphs of the functions.

2. Important properties of the limits must be revising to students to use them directly to minimize the calculation.

3. While solving the differentiation of implicit functions, students commits the mistakes in differentiating dependent variable simply, they don't apply chain rule while differentiating dependent variable. For example ,
$$x^2 + y^2 = y$$
They differentiate it wrongly as : $2x + 2y = \frac{dy}{dx}$
Practice of several questions makes them to do calculation free from such errors.

4. While differentiating the logarithmic function, the students make very common and frequent mistakes while applying the properties of logarithmic.
   For example if we are given the function of the type
   $y = x^{\sin x} + (\sin x)^{\cos x}$

   Then students take the log on both sides wrongly in the following way,
   $$\log y = \log\left(x^{\sin x} + (\sin x)^{\cos x}\right)$$
   $$\log y = \log x^{\sin x} + \log(\sin x)^{\cos x}$$
   This is absolutely wrong.
   To avoid such mistakes, we as a teacher must have to take a class that is specially dedicated on logarithmic as there is no dedicated chapter in the syllabus on this up to class 12. This will help them not only in mathematics but also in physics and chemistry.

5. Apart from all above, the students commits such unexpected mistakes, which could not be framed. Mistakes are parts of study, but as a teacher we have to make them aware of each and every mistake in a very positive way which will be an ultimate remedy of all mistakes. We have to guide them in area of their need above and par our capacity. Practice of questions of the area of their mistakes will definitely make them better day by day.

Chapter - 6
APPLICATION OF DERIVATIVE

TOPIC	COMMON ERRORS/MISTAKES	REMEDIES
Increasing/Decreasing Functions	• Incorrect identification of the intervals after obtaining the critical points. • Incorrect sign of f'(x) to identify the increasing /decreasing functions • Incorrect differentiation of the given functions. • Incorrect factorization of algebraic function to obtain the critical points. • Finding intervals for T-functions involving multiples and sub multiples of angles.	• After obtaining the critical points it is better to draw the table for identifying the sign of f'(x) in different intervals. • Correctly observing the sign of f'(x) in the interval before deciding the nature of the function. • Revising the question after solving. • ASTC should be followed.
Tangents and Normal	• Incorrect slope of the tangent • Confusion between slope of tangents and normal. • Incorrect equation of tangent due to wrong entry in the equation of line. • Wrong interpretation of the questions.	• Be alert while taking the slope of normal as it is negative multiplicative inverse of the slope of tangents. • Entries of values should be correctly made while writing the equation of tangents/normal.
Maxima and Minima	• Incorrect function differentiated to obtain Maxima/Minima • Incorrect conversion of function as it is to be converted in terms of any one variable with respect to whom it is to be differentiated to obtain critical points. • Wrong interpretation of the questions • Problem in understanding the question • Difficulty in observing the function which is to be maximize /minimize.	• Correct function to be identified which is to be maximize/minimize before differentiating. • Sign of f"(x) to be properly observed before declaring any function to be having point of maxima/minima. • Always try to identify the function which is to be maximize/minimize before differentiating.

Chapter - 7
INTEGRALS

Common errors/mistakes commited by the students and their remedies.

1. Forgetting to put the constant of integration.

Goal: $\int x^3 dx$

Mistake: $\int x^3 dx = \frac{x^4}{4}$

Correction: $\int x^3 dx = \frac{x^4}{4} + c$

Explanation: some students forget to put the constant of integration after that your 1/2 mark cut in examination. So don't forget to put the constant of integration.

2. Mixing differentiation with integration (especially in formula)

1) **Goal:** $\int \boldsymbol{sinxdx}$
The Mistake: :$\int sinxdx = cosx + c$
The Correction: $\int sinxdx = -cosx + c$
An Explanation
The integral of the sine function is not cosine, but rather is negative cosine, since the *derivative* of cosine is *negative* sine.

2) **The Goal** : Find $\int \boldsymbol{cosxdx}$
The Mistake: Find the mistake: $\int \boldsymbol{cosxdx} = \boldsymbol{-sinx + c}$
The Correction: $\int \boldsymbol{cosxdx} = \boldsymbol{sinx + c}$

An Explanation: The integral of the cosine function is not negative sine, but rather is positive sine, since the derivative of sine is cosine.

3) The Goal: find $\int \boldsymbol{e^x dx}$
The Mistake : Find the mistake:

$$\int e^x\,dx = \frac{1}{x+1}e^{x+1} + C$$ (NOTED)

The Correction: $\int \boldsymbol{e^x dx = e^x + c}$
An Explanation: An exponential function is *not* a power function, so the integral formula for a power function does *not* apply to finding the integral in this example.

3. Integration by parts: (PROPER CHOICE OF FIRST AND SECOND FUNCTION)

Find :$\int \boldsymbol{xsinxdx}$

The Mistake: :$\int xsinxdx = 1.cosx + c$

Correction : $\int xsinxdx = x.\int sinxdx - \int[\int sinxdx \frac{d}{dx}x]dx$

= $x.(-cosx) - \int(-cosx).1dx$

$= -x.cosx + sinx + c$

An Explanation: some students use product rule of differentiation but in such type question , use by parts method using "ILATE" where I stands inverse trigonometric functions, L stands logarithmic functions, A stands algebraic functions , T stands trigonometric functions and E stands exponential functions. We can also choose the first function as the function which come first in the word "ILATE" .

4. **SUSTITUTION METHOD(PROPER USE)**

1) Find :$\int \boldsymbol{sin(2x)\, dx}$

The Mistake: :$a)\ \int \sin(2x)\, dx = -cos2x + c$, b) $\int \sin(2x)\, dx = -2cos2x + c$

Correction :$\int \sin(2x)\, dx = -\frac{1}{2}cos2x + c$,

2) Find :$\int \frac{\boldsymbol{4X}}{\sqrt{\boldsymbol{x^2+1}}}\boldsymbol{dx}$.

The Mistake: Find mistake $I = \int \frac{\boldsymbol{4X}}{\sqrt{\boldsymbol{x^2+1}}}\boldsymbol{dx}$

Put $\boldsymbol{x^2+1=t}$ so, 2xdx=dt

$\mathrm{I} = 2\int \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{t}}}\boldsymbol{dt} = 2\sqrt{\boldsymbol{t}}.\boldsymbol{2}+\boldsymbol{c} = \boldsymbol{4}\sqrt{\boldsymbol{t}}+\boldsymbol{c}$ **(noted)**

Correction: **Put $\boldsymbol{x^2+1=t}$ so, 2xdx=dt**

$$\mathrm{I} = 2\int \frac{\boldsymbol{1}}{\sqrt{\boldsymbol{t}}}\boldsymbol{dt} = 2\sqrt{\boldsymbol{t}}.\boldsymbol{2}+\boldsymbol{c} = \boldsymbol{4}\sqrt{\boldsymbol{t}}+\boldsymbol{c} = \boldsymbol{4}\sqrt{\boldsymbol{x^2+1}}+\boldsymbol{c}$$

Explanation: some students not replace t in x . So, remember it

5. **Limit of definite integration :**

1) Find :$\int_0^1 2x(x^2+3)^{\frac{1}{2}}dx$

The Mistake: Find mistake $\boldsymbol{I} = \int_0^1 2x(x^2+3)^{\frac{1}{2}}dx$

Put $\boldsymbol{x^2+3=t}$ so, 2xdx=dt

$\boldsymbol{I} = \int_0^1 (t)^{\frac{1}{2}}dt$ (noted)

$$= \left[\frac{2}{3}(t)^{\frac{3}{2}}\right]_0^1 = \frac{2}{3}$$

Correction: Put $\boldsymbol{x^2+3=t}$ so, 2xdx=dt

$\boldsymbol{I} = \int_3^4 (t)^{\frac{1}{2}}dt$ (noted limits change)

$= \left[\frac{2}{3}(t)^{\frac{3}{2}}\right]_3^4 = \frac{2}{3}(8-3\sqrt{\boldsymbol{3}}\,)$

Explanation: some students not change the limits of t . So, remember it

2) The Goal: find $\int_0^2 e^{-x}\,dx$

The Mistakes

Find the mistakes:

1. $\int_0^2 e^{-x}\,dx = e^{-x}|_0^2 = e^{-2} - e^0 = e^{-2} - 1$

2. $\int_0^2 e^{-x}\,dx = -e^{-x}|_0^2 = -e^{-2} + (-e^0) = -e^{-2} - 1$

A Correct Solution: $\int_0^2 e^{-x}\,dx = -e^{-x}|_0^2 = -e^{-2} - (-e^0) = -e^{-2} + 1$

Explanations

In the first mistake the ant derivative is computed incorrectly - the negative sign is needed.

In the second mistake a common sign error is made. If F(x) is an ant derivative of the integrand (the function in the integral), the value of the definite integral from a to b is F(b)-F(a), which applies even if F(x) has a minus sign in front of it. So if $F(x) = -e^{-x}$, then $F(b)-F(a) = (-e^{-b}) - (-e^{-a}) = e^{-a} - e^{-b}$.

6. **Proper use of the formula for $\int x^n dx$**

Many students forget that there is a restriction on this integration formula, so for the record here is the formula along with the restriction.

$\int x^n dx = x^{n+1}/(n+1)+c$, provided $n \neq -1$

That restriction is incredibly important because if we allowed $n=-1$ we would get division by zero in the formula! Here is what I see far too many students do when faced with this integral.

$\int x^{-1}dx = x^0/0 + c = x^0 + c = 1 + c$

There are all sorts of problems with this. First there's the improper use of the formula, then there is the division by zero problem! This should NEVER be done this way.

Recall that the correct integral of x^{-1} is,

$\int x^{-1}dx = \int \frac{1}{x}dx = \log|x| + c$

This leads us to the next error.

7. **Dropping the absolute value when integrating $\int \frac{1}{x}\,dx$**

Recall that in the formula

$\int \frac{1}{x}dx = \log|x| + c$

the absolute value bars on the argument are required! It is certainly true that on occasion they can be dropped after the integration is done, but they are required in most cases. For instance, contrast the two integrals,

1. $\int \frac{2x}{x^2+10}dx = \log|x^2+10| + c$
2. $\int \frac{2x}{x^2-10}dx = \log|x^2-10| + c$

In the first case the x^2 is positive and adding 10 on will not change that fact so since $x^2+10>0$ we can drop the absolute value bars. In the second case however, since we don't know what the value of x is, there is no way to know the sign of x^2-10 and so the absolute value bars are required.

8. **Improper use of the formula $\int \frac{1}{x}dx = \log|x| + c$**

Gotten the impression yet that there are more than a few mistakes made by students when integrating $\frac{1}{x}$? I hope so, because many students lose huge amounts of points on these mistakes.

In this case, students seem to make the mistake of assuming that if $\frac{1}{x}$ integrates to log|x| then so must one over anything! The following table gives some examples of incorrect uses of this formula.

Integral Incorrect Answer Correct Answer

$\int\frac{1}{x^2+1}dx$	$\log(x^2+1)+c$	$\tan^{-1}(x)+c$
$\int\frac{1}{x^2}dx$	$\log(x^2)+c$	$-x^{-1}+c=-1/x+c$
$\int\frac{1}{\cos x}$	$\log(\cos x)+c$	$\log\lvert\sec x+\tan x\rvert+c$

So, be careful when attempting to use this formula. This formula can only be used when the integral is of the form $\int\frac{1}{x}dx$. Often, an integral can be written in this form with an appropriate u-substitution (the two integrals from previous example for instance), but if it can't be then the integral will NOT use this formula so don't try to.

9. Improper use of Integration formulas in general

This one is really the same issue, but so many students have trouble with logarithms that I wanted to treat that example separately to make the point.

So, for instance we've got the following two formulas,

$\int\sqrt{\boxed{u}}\,du=\frac{2}{3}u^{\frac{3}{2}}+C$

$\int u^2du=1/3u^3+C$

The mistake here is to assume that if these are true then the following must also be true.

$\int\sqrt{\boxed{anything}}\,du=\frac{2}{3}(anything)^{\frac{3}{2}}+C$

$\int(anything)^2du=1/3(anything)^3+C$

The first set of formulas work because it is the square root of a single variable or a single variable squared. If there is anything other than a single u under the square root or being squared then those formulas are worthless. On occasion these will hold for things other than a single u, but in general they won't hold so be careful!

Here's another table with a couple of examples of these formulas not being used correctly.

Integral	Incorrect Answer	Correct Answer
$\int\sqrt{\boxed{x^2+1}}\,dx$	$2/3(x^2+1)^{3/2}+C$	$\frac{1}{2}(x\sqrt{x^2+1}+\log\lvert x+\sqrt{x^2+1}\rvert)+C$
$\int\cos^2xdx$	$1/3\cos^3x+C$	$x/2+1/4\sin(2x)+C$

10. Loss of integration notation

1) There are many dropped notation errors that occur with integrals. Let's start with this example.

$\int x(3x-2)dx=3x^2-2x = x^3-x^2+C$

Here, drop the integral sign you are saying that you've done the integral.

But Here is the correct way to work this problem.

$\int x(3x-2)dx=\int(3x^2-2x)dx=x^3-x^2+c$

2) Another big problem in dropped notation is students dropping the dx at the end of the integrals. For instance,

$\int 3x^2-2x$

The problem with this is that the dx tells us where the integral stops! So, this can mean a couple of different things.

$\int(3x^2-2x)dx=x^3-x^2+c$ OR $\int 3x^2dx-2x=x^3-2x+c$

Without the dx a reader is left to try and intuit where exactly the integral ends! The best way to think of this is that brackets always come in pairs "(" and ")".

3) Another dropped notation error that I see on a regular basis is with definite integrals. Students tend to drop the limits of integration after the first step and do the rest of the problem with *implied* limits of integration as follows.

$\int_1^2 x(3x-2)dx=\int(3x^2-2x)dx=x^3-x^2=(8-4)-(1-1)=4$

The answer to a definite integral is a number, while the answer to an indefinite integral is a function. When written as above you are saying the answer to the definite integral and the answer to the indefinite integral are the same when they clearly aren't!

Here is the correct way to work this problem.

$$\int_1^2 x(3x-2)dx = \int_1^2 (3x^2-2x)dx = (x^3-x^2)_1^2 = (8-4)-(1-1) = 4$$

11. Dropped constant of integration

Dropping the constant of integration on indefinite integrals (the +c part) is one of the biggest errors that students make in integration. There are actually two errors here that students make. Some students just don't put it in at all, and others drop it from intermediate steps and then just tack it onto the final answer.

So don't forget the constant of integration on indefinite integrals (the +c part) in the final answer.

- In indefinite integral students forget to write "constant".
  Remedy:- It is must to write "constant" in indefinite integral so students should remember this.
- Some time students write more than one constants when they do integration of terms in one problem separately. **Remedy:-** This is not false but it may create problem in finding particular result. So students should write "constant" once at last of the final answer.Otherwise they should combined all constants in only one constant.
- Students have a view that in integration "x" is only the variable of integration. **Remedy:-** When the variable of integration is denoted by a variable other than "x" the integral formula are modified accordingly.
  $\int y^4 dy = \frac{y^{4+1}}{4+1} + c = \frac{y^5}{5} + c$. Here variable is "y".
- Students drop absolute value sign in integration of the form $\int \frac{1}{ax+b} dx$.
  Remedy:-The absolute value sign can sometimes ignore but this is required in most cases.so don't avoid this sign in integration of this given form. Firstly use this sign and if not necessary then it can avoid otherwise not.
- Improper use of the formula $\int \frac{1}{x} dx = ln|x| + c$. Students do mistake in using this formula.
  Remedy:-always remember that formula is applicable only when denominator has only linear form of variable like (ax + b) type, otherwise this formula will not work.
- Due to relation with differentiation students forget formula of integration especially sign problem occurs mostly.
  Remedy:- for this learn formula properly and do more practice on integral problems.
- Students confuse in solving partial fraction related integral problems.
  Remedy:- Students should remember all cases of partial fraction and more practice of related problems will help them to find which partial fraction type is applicable in given problem.
- Students confuse in solving integration by parts related problems which should be taken first or which should be taken second?
  Remedy:- In integration by parts we can use this word for selecting first and second function on the basis of their order from left to right.
  "ILATE" .
- In definite integral students solve problem directly due to this many times problem becomes lengthy.
  Remedy:-students should remember properties of definite integral and should use these properties in their problems.it will help them to solve problem in easy way.

12. Many students think $sin^{-1}x and (\sin x)^{-1}$ both are same. Same they think for others trigonometric functions.
Remedy:- Both are not same we have to clarify students difference between inverse trigonometric functions and trigonometric functions with powers.so that they can use correct formula of integration

Types of Mistake	Concept	Probable errors	Action taken by teachers
1. Misconceptions 2. Procedural errors 3. Integration techniques 4. Learning difficulties 5. Understanding the problems (Specially the problems related to Application of Integrals)	1- Integration as an Inverse Process of Differentiation. 2- Integration by Substitution. 3-Integration using Trigonometric Identities. 4- Integral of some Particular Functions. 5- Integration by Partial Fractions. *6- Integration by Parts* 7-Deefinite Integral. 8- Evaluation of Definite integral by Substitution. 9-Some properties of Definite Integrals. integration formulae substitution method Making perfect square constant of integration computation while evaluating the definite integral	1.Students get confused with differentiation and integration formulae. a) Students committed mistakes in putting correct sign of anti-derivatives. b) Feel difficulty in learning ant derivatives of some functions a^x. 2. *In substitution method students find it difficult to substitute correctly. * Students feel more difficulty in double substitution. *Students make wrong substitution ultimately unable to do solve the problem. 3- *Students apply wrong trigonometric formula ultimately problem could not be solved. Event not learning formulae. 4- While doing the problems based on some particular function, students feel more difficulty in converting function in standard form to apply formula, specially in making perfect square. 5- Some time students committed mistakes in calculating values of constants in partial Fractions. 6- Students feel difficulty in selecting first function while doing the problems of integration by parts. 7- Students make silly mistakes in calculating	1. *Conduct formula practice test daily - oral and written. *To motivate students to learn formula and solve problems using formulae * Practice of repeatedly writing formulae by students before teacher. 2-*Teacher should give the problems involving variety of substitution. *Teacher make drilling the correct substitution of dx. * To provide some easy problems on double substitution. 3- Teacher should provide variety of problems using different trigonometric identities. 4- Teacher should ensure more practice in such problems, especially in making perfect square. *Teacher should set a pattern in learning these formulae, and solving such problems. 5- Teacher should make drill on it. Taking variety of problems based on partial fractions. 6- Teacher should give some tips regarding selecting first function. ILATE rule etc. 7- teacher should advised students to not make such mistakes, to avoid wastage of time in recalculating the definite integral.

		values of definite integral. 8- Students feel problems in changing limits while making substitution in definite integral. Either they don't change limit or making mistake in it. 9- Students committed frequent mistakes while evaluating definite integrals using properties of definite integrals. 10- students fail to identify the method, which they have to opt. 11- Making perfect square while doing the problems of the type $\int dx / (ax^2 + b x + c)$ 12 students miss the constant of integration while writing the answer & forget to use dx while integrating.	8- Teacher should ensure more drill on it by taking different problems on it. Teacher should solve some problems before students specially related to change of limits while making substitution. 9- Teacher should advised to students to make more drill on it. Teacher should give some tips to students to solve such problems involving long calculations. 10- Classification of problems based on different methods using different formulas should be stressed. 11 Drilling up the same method and conduct slip test frequently. 12-. Concept should be made clear of putting C, while writing integrals of indefinite integrals.

Chapter – 8
APPLICATION OF INTEGRALS

S.NO.	ERRORS	CORRECTIONS
1	Students are unable to identify whether the given equation is a straight line, circle, parabola, ellipse etc	Students should do practice on the recognition of equations of different types of curves.
2	Students are not able to draw the rough graph of the given equation.	Students should do practice to draw the graph of the various functions.
3	Students make mistake in shading the required area.	Students should read the question carefully and shade the asked area.
4	Students skip the steps during calculation and unable to reach the answers.	Students should not skip the steps and do the calculations carefully.
5	Students do not write the unit.	Students should write the sq. units with the answer.

Remedial Measures for Under Achievers

1. Identify the area in which under achievers need help.
2. Instead of the compete syllabus selected subject material will be given to them.
3. Pair with one bright student with under achievers so that he/she can improve his/her performance.
4. Ask under achievers to frame short questions and conduct a test on the basis of those questions only to increases confidence.
5. Conceptual problems should be discussed with him and ask him to solve on board.
6. Content material will be developed according to their ability.
7. Teaching methodology will be applied according to their based requirement.
8. Keep in touch with the parent of under achievers regarding their performance.
9. The under achievers should be encouraged by way of effective motivation in the form of rewards and praise.
10. Discussion with teachers should be encouraged.
11. To improve the memory level of under achievers meditation, exercises should be arranged.
12. Focus on the most common questions asked in CBSE board exam.
13. Parent counseling and student's counseling
14. Peer learning should be encouraged.

Chapter - 9
DIFFERENTIAL EQUATIONS

Common errors/mistakes committed by the students and their remedies -

S No	Common errors/mistakes committed by the students	Remedies
1.	Difference between order and degree	Degree on higher order derivative will be the degree of the differential equation. Explain by giving some examples.
2.	In finding the degree of a differential equation when it is not defined.	When a differential equation is not a polynomial in derivatives its degree is not defined. More practice and understanding of questions in which degree is not defined.
3.	Difference between General solution and Particular solution	General solution contains an arbitrary constant where as particular solution contains particular value in place of arbitrary constant.
4.	Confusion in finding differential equation from given solution	Form the differential equation by eliminating arbitrary constants from the given relation ie order of differentiation is equal to number of arbitrary constants
5.	Separation of variables	Arrange dx and dy separately along with its variables.
6.	Confusion in value of P and Q in Linear differential equation	$\frac{dy}{dx}$ + P(x)y = Q(x), coefficient y will be the value of P. P and Q both are the function of x only
7	Integrating factor	Integrating factor (IF)= e$\int P(x)dx$ Integrate P with respect to x and put on the power of e.
8	Not writing constant of integration in solutions.	As marks are deducted for not writing constant of integration. Always remember to write constant of integration while integrating.
9	Not finding the value of C (constant of integration) in the questions of finding particular solution of a differential equation.	More practice of questions of finding the particular solution.
10	Solving homogeneous differential equations using method for solving linear differential equations.	Understanding to differentiate between two types of differential equations .more practice to solve homogeneous differential equations.
11	Unable to solve the homogeneous differential equation if they are of the form: dx/dy = f(x/y)	More practice of such type of questions.
12	• Students fail to identify the type of differential equation • Students apply incorrect method to solve the differential equations when they fail to identify the type of differential equation • Students apply incorrect method to integrate the differential equations	• Drilling up different types of differential equations problems based on different methods • Giving sufficient number of problems for practice • Concept should be made clear so that they can differentiate between homogeneous and linear differential equations

Chapter - 10
VECTOR ALGEBRA

Common Mistakes occurred during solving problems of Vector Algebra

1-Collinear Vector: Show that the points $A(-2\hat{\imath}+3\hat{\jmath}+5\hat{k}),\ B(\hat{\imath}+2\hat{\jmath}+3\hat{k}) and\ C(7\hat{\imath}-\hat{k})$ are collinear.

Common Mistake $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

Correct form: $|\overrightarrow{AB}|+|\overrightarrow{BC}|=|\overrightarrow{AC}|$

2-Triangular Inequality; For any two vectors $\vec{a}\ and\ \vec{b}$

Common Mistake: $|\vec{a}+\vec{b}|=|\vec{a}|+|\vec{b}|$

Correct Form : $|\vec{a}+\vec{b}|\le|\vec{a}|+|\vec{b}|$

If $|\vec{a}+\vec{b}|=|\vec{a}|+|\vec{b}|\ then\ points\ are\ collinear$

3-Equality of two vector: Find the values of *x*, *y* and *z* so that the vectors $\vec{a}=x\hat{\imath}+2\hat{\jmath}+z\hat{k}, and\ \vec{b}=2\hat{\imath}+y\hat{\jmath}+\hat{k}$

Common Mistake: $|\vec{\boldsymbol{a}}|=|\vec{\boldsymbol{b}}|$

Correct Form: two vectors are equal then corresponding component equal

$$x\hat{\imath}+2\hat{\jmath}+z\hat{k}=2\hat{\imath}+y\hat{\jmath}+\hat{k}$$

X=2,y=2,z=1

4- Projection of Vector: Find the projection of the vector $\vec{a}=2\hat{\imath}+3\hat{\jmath}+2\hat{k}$ on the vector $\vec{b}=\hat{\imath}+2\hat{\jmath}+\hat{k}$

Common Mistake: Simply multiply $\vec{a}.\vec{b}=(2\hat{\imath}+3\hat{\jmath}+2\hat{k}).(\hat{\imath}+2\hat{\jmath}+\hat{k})$

Some time only add the vectors ; $\vec{a}+\vec{b}$

Correct Form : Projection of $\vec{a}\ on\ \vec{b}=\frac{\vec{a}.\vec{b}}{|\vec{b}|}$ and

Projection of $\vec{b}\ on\ \vec{a}=\frac{\vec{a}.\vec{b}}{|\vec{a}|}$

5- Vector in direction of other Vector: Find the vector of magnitude 7 in the direction of the sum of the vectors ,$\vec{a}=2\vec{\imath}+2\vec{\jmath}-5\vec{k}\ and\ \vec{b}=2\hat{\imath}+\hat{\jmath}+3\hat{k}$

Common Mistake:: vector of magnitude 7 = $7(\vec{a}+\vec{b})$ =7{$(2\vec{\imath}+2\vec{\jmath}-5\vec{k})$+$(2\hat{\imath}+\hat{\jmath}+3\hat{k})$}

Correct Form: Write unit vector in the desired direction then multiply by 7

vector of magnitude 7 = $7\frac{(\vec{a}+\vec{b})}{|(\vec{a}+\vec{b})|}$ = $7\frac{4\vec{\imath}+3\vec{\jmath}-2\vec{k}}{\sqrt{4^2+3^2+-2^2}}$.

=$7\frac{4\vec{\imath}+3\vec{\jmath}-2\vec{k}}{\sqrt{29}}$

6- Vector Product: Find a unit vector perpendicular to each of the vectors $(\vec{a}+\vec{b})\ and\ (\vec{a}-\vec{b})$ where $\vec{a}=\vec{\imath}+\vec{\jmath}+\vec{k}$ and $\vec{b}=\hat{\imath}+\hat{2\jmath}+3\hat{k}$

Common Mistake: Generaly show that $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})$

Correct Form: $\vec{a}+\vec{b})=2\vec{\imath}+\overline{3\jmath}+4\vec{k}$ and $(\vec{a}-\vec{b})=-\vec{\jmath}-2\vec{k}$

A vector which is perpendicular to both is given by $(\vec{a}+\vec{b}).x(\vec{a}-\vec{b})$=$\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 3 & 4\\ 0 & -1 & -2\end{vmatrix}$

$$=-2\hat{\imath}+4\hat{\jmath}-2\hat{k}=\vec{c}(let)$$

$|\vec{c}|=\sqrt{24}$

Therefore, the required unit vector is $\frac{\vec{c}}{|\vec{c}|}=\frac{-2\hat{\imath}+4\hat{\jmath}-2\hat{k}}{\sqrt{24}}$

7-Area of triangle when two side vectors Given: Find the area of a triangle having the points A(1, 1, 1), B(1, 2, 3) and C(2, 3, 1) as its vertices

$\overrightarrow{AB}=\hat{\jmath}+2\hat{k}$

$\overrightarrow{AC}=\hat{\imath}+2\hat{\jmath}$

Commmon Mistake: Area = $\frac{1}{2}\overrightarrow{AB}x\overrightarrow{AC}$ (Generally leave at this stage)

Corroect Form: $\frac{1}{2}|\overrightarrow{AB}x\overrightarrow{AC}|$ **(Scalar quantity)**

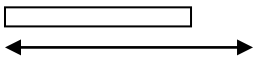
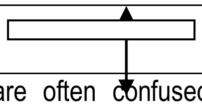
$$\frac{1}{2}|(\hat{j}+2\hat{k})x(\hat{\imath}+2\hat{j})|$$

$$\overrightarrow{AB}x\overrightarrow{AC}=\begin{vmatrix}\hat{\imath} & \hat{j} & \hat{k}\\ 0 & 1 & 2\\ 1 & 2 & 0\end{vmatrix}=-4\hat{\imath}+2\hat{j}-\hat{k}$$

$$\frac{1}{2}|\overrightarrow{AB}x\overrightarrow{AC}|=\frac{1}{2}\sqrt{21}$$

Concept	Error	Correction
Finding a vector of given magnitude in the direction of another vector	Forget to find the unit vector in the direction of given vector	Stress on finding the magnitude of the given vector
Finding an unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$	Use the formula as $\frac{\vec{a}\times\vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$	Stress on to use the correct formula
Scalar product of two vectors	After using formula of dot product use of $\hat{\imath},\hat{j},\hat{k}$	In final result of scalar product of two vectors we get a scalar quantity so not to use $\hat{\imath},\hat{j},\hat{k}$
Area of triangle and parallelogram	Mistakes of not putting or putting ½ with $\lvert\vec{a}\times\vec{b}\rvert$	Stress on to use the correct formula
	• Misunderstand the direction of vectors. • Confusion in scalar components and vector components. • Confusion in dot(scalar) product and (cross)vector product formulae. • Confusion in i.i,j.j ,k.k and i×i , j×j and k×k • Misunderstand the direction of vector product $\vec{a}\times\vec{b}$.	More emphasis must be given on basic concepts, formulae and key points. -Regular practice and more practice. - Memorize in own language.

Chapter - 11
THREE DIMENSIONAL GEOMETRY

S No	Area where mistake committed	Detail about mistakes committed	Remedy
1	d.c.s and d.r.s	Students sometimes do not understand difference between them,	Clearly they should be explained that For d.r.s ; a,b,c $a^2+b^2+c^2 \neq 1$ for d.c.s l,m,n $l^2+m^2+n^2=1$
2	Standard Equation of a straight line	Student directly starts to solve problem in equation like $\frac{2x-1}{2}=\frac{4-y}{7}=\frac{z+1}{2}$	First they should be explained the standard form $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ Then tell them to convert the given equation in standard form.
3	Plane parallel to a straight line	Plane Line 	But here condition of perpendicularity will be used
4	Plane perpendicular To a straight line	Plane Line 	But here condition of parallelism will be used
5	Equation of plane to be used in given problem	Students are often confused for equation of plane to be used in question	Students should be asked to focus on the language and get sense from there
6	Line lies on a plane	When line $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ lies on the plane $a_1x+b_1y+c_1z+d=0$ then what will happen	All the things related to this concept should be explained to the students.
7	.Shortest distance between two parallel lines	Students Apply the formula of shortest distance between two skew lines.	Concept should be clear to students through problem solving
8	Use of formulae of 3 d	students fail to identify the method , which they have to opt	Classification of problems based on different methods using different formulas should be stressed

Chapter - 12
LINEAR PROGRAMMING

Wealth/Strength of subject ability and how to remove general error.
Notes :-

1. Read the question carefully, understand each statement.
2. Note that which quantities do we require to obtain for maximum, or minimum. Represent those quantities by the symbols x and y. Note $x \ge 0$, $y \ge 0$.
3. Form the inequations involving x and y from the given conditions.
4. Some students are not able to apply the inequality signs correctly. Represent the inequations with sign
   (i) $\ge$, if it is mentioned at least, greater than equal to, minimum of, in the given conditions.
   (ii) $\le$, if it is mentioned at most, less than equal to, maximum of, in the given conditions.

Question	**some students write it as :**	**correct solution/statement is :**
Shade the portion for the inequations $x \ge 0$, $y \ge 0$, $4x+3y-12 \le 0$	After drawing the line $4x+3y-12=0$ Put (1,1) in $4x+3y-12 \le 0$ $4+3-12 \le 0$ $-5 \le 0$, False We shade the portion not containing (1,1) which is a WRONG statement	After drawing the line $4x+3y-12=0$ (i) For the inequation $4x+3y-12 \le 0$ Take any point which does not lie on the line. Generally we take (0,0), if it does not lie on the line. Here we put (0,0) ; $0+0-12 \le 0$ $-12 \le 0$, True We shade the portion containing (0,0) Also $x \ge 0$, $y \ge 0$
A firm manufactures two types of products A and B and sells them at a profit of Rs.5 per unit of type A and Rs.3 per unit of type B. Each product is processed in two machines M1 and M2. One unit of type A requires 1 minute processing on M1 and 2 minutes on M2. Whereas 1 unit of type B requires 1 minute of processing time on M1 and 1 minute on M2. Machines M1 and M2 are respectively available for atmost 5 hours and 6 hours in a day. Find out how many units of each type of product should the firm produce in a day in order to maximize the profit. Solve the problem graphically.	Let x units of product A and y units of product B are produced per day. Then LPP is To maximize $Z = 5x+3y$ subject to constraints $x \ge 0$, $y \ge 0$ $x+y \le 5$ $2x+y \le 6$ Above both the statement are WRONG statements. [Note that 5 and 6 are in hours]	let x units of product A and y units of product B are produced per day. Then LPP is To maximize $Z = 5x + 3y$ subject to constraints $x \ge 0$, $y \ge 0$ $x+y \le 300$ $2x+y \le 360$ Now find the the feasible region and proceed

sometime a question is "……….one unit of product A requires 2 minutes for grinding and 3 minutes for cutting and one unit of product B requires 1 minute for cutting and 3 minutes for grinding. Each machine, for grinding and cutting is available for atmost 4 hours in a day. Find……….”	Let x units of product A and y units of product B are produced per day. Then LPP is To maximize $Z = 5x+3y$ (say) subject to constraints $x\geq 0, y\geq 0$ $2x+y \leq 240$ $3x+3y \leq 240$ Above both statement are WRONG. [In this question note the placements of cutting/grinding]	Let x units of product A and y units of product B are produced per day. Then LPP is To maximize $Z = 5x+3y$ subject to the constraints $x\geq 0, y\geq 0$ $2x+3y \leq 240$ $3x+y \leq 240$ Now find the feasible region and proceed

Common error committed by students are as follows:

1: Symbols used in linear programming problem that is, <, >, ≤, and ≥.
2: Making of objective function.
3: wrong interpretation of problems.
4: Simplification of linear equation.
5: Drawing of lines by wrong coordinates.
6: choosing of wrong feasible region.
7: Posting mistakes
8: difference of bounded and unbounded feasible region.
9: **Changing the Inequality Sign**.

Remedies to remove errors:
1: Developing and using symbol sense in mathematics.
2: while solving the Problems must interpret the problems, so that to reduce the understanding of question.
3: Try to take the co-ordinates on the axes by substituting x=0 in one time and y=0 in other case.
4: Always take $x \geq 0$, $y \geq 0$.
5: verify the feasible regions by taking suitable points.
6: taking proper scale for drawing lines.
7: To avoid posting mistakes calculation must be on the same page by making a margin.
8: When multiplying/dividing any inequality by –1, the direction of the inequality should change.

Chapter - 13
PROBABILITY

Concept/topic	Probable errors	Action to be taken
Conditional probability	Unable to identify the conditional event in questions based on ***'conditional probability'***	More practice of such questions
Identifying the types of events	Unable to identify the question (whether conditional or independent events or Bayes' theorem)	More practice of questions of various types .
Mathematical findings from Word problems	Difficulty in converting word problem into mathematical terms	Drilling in conversion of different kinds of problems
***Conditional events in* Bayes' theorem**	Mistakes in identifying different 'EVENTS' in Bayes' theorem Mistakes in identifying the probability of different events in Bayes' theorem	More practice of such questions Drilling in such problems
Computation	Computational mistakes	More concentration and attention
Probability distribution	Inability to find out the correct random variable Unable to form the probability distribution table	More practice of such questions More practice to be given
Final value	Forget to write the final answer	Emphasis on writing the final answer. Reminding again and again

COMMON MISTAKES/ERRORS COMMITTED BY STUDENTS IN TOPIC : PROBABILITY

1. To score well in the Topic "Probability", the students must have clear concepts of sets, operations on sets, and better understanding of the terms : random experiment, outcomes, sample space, trials, events, types of events, probability, conditional probability, independent events, partition of sample space, probability distribution etc. They should also properly understand the multiplication theorem on probability, theorem of total probability and Bayes' theorem etc. Most of the common mistakes done by students can be overcome with proper understanding of basic concepts.

2. The students, many times, put the probability of event A i.e. P(A), in the denominator, in the result for conditional probability P(A/B), whereas correct result is $P(A/B)=\dfrac{P(A\cap B)}{P(B)}$

3. They, many times, forget how to find the probability of not happening of event A. The correct result is $P(A')=1-P(A)$

4. It is very important to read the question carefully, the students generally miss the term "with replacement" or "without replacement" while doing problems of "taking out some articles from a bag".

5. The students should have clear knowledge of "how many" and "what type of cards" constitute "a pack of playing cards"

6. The students should use the result of “conditional probability” to understand the “multiplication theorem on probability” and to do the problems using concepts, not just by cramming the results. The correlation of concepts is very important.

7. The students should be able to clearly distinguish between “independent events” and “mutually exclusive events”. They should know how to check two or more events as independent .

8. Instead of cramming the result for Bayes’ theorem, the students should have clear understanding of partition of sample space, theorem of total probability and Bayes’ theorem step by step and to clearly find out hypotheses, priori and posteriori probability of hypothesis, by reading the given question carefully, and then only put the values in the result for Bayes’ theorem. They should also carefully find out whether the given question is of “Theorem of total probability” or that of “Bayes’ theorem”

MINIMUM LEARNING MATERIAL
(CAPSULE FOR UNDERACHIEVERS)

Chapter -1
RELATIONS AND FUNCTIONS

KEY POINTS:

1. **A relation R in set A is called an equivalence relation in set A if it is**
   (a)reflexivei.e (x,x)ϵR , $\forall x \epsilon A$
   (b) symmetrici.e if (x,y) ϵR =>(y,x)ϵR , $\forall x, y \epsilon A$
   (c)transitivei.e if (x,y) ϵR &(y, z) ϵR =>(x,z)ϵR , $\forall x, y, z \epsilon A$

2. **n(A)= m, then number of relation in A is 2^{m^2}. Number of reflexive relation $2^{m^2} - m$.**

3. **A function f:A→ B is invertible(bijective) if it is (a) one-one (injective) i.e $f(x_1)=f(x_2) \Rightarrow x_1=x_2$, $x_1,x_2 \epsilon A$, (b) onto(surjective) i.e f(x)=y defined for all y $\epsilon codomain$ i.e. Range= Codomain**

4. **y= f(x) and f is BijectiveFuntion, then It is invertible and x=f^{-1}(y) .**

Objective Type Questions

1. If A = {1, 2, 3}, then the relation R = {(1, 2), (2, 3), (1, 3)} in A is ___.
   a. transitive only
   b. reflexive only
   c. symmetric only
   d. symmetric and transitive only

2. A relation R on a set A is called an empty relation if
   a. no element of A is related to any element of A
   b. every element of A is related to one element of A
   c. one element of A is related to all the elements of A
   d. every element of A is related to any element of A

3. Let R be the relation on N defined as xRy if x + 2 y = 8. The domain of R is
   a. {2, 4, 6, 8}
   b. {2, 4, 8}
   c. {1, 2, 3, 4}
   d. {2, 4, 6}

4. Consider the non-empty set consisting of children in a family and a relation Rdefined as*a*R*b*if *a* is brother of *b*. Then R is
   (A) symmetric but not transitive (B) transitive but not symmetric
   (B) neither symmetric nor transitive (D) both symmetric and transitive

5. The maximum number of equivalence relations on the set A = {1, 2, 3} are
   (A) 1 (B) 2
   (C) 3 (D) 5

6. If a relation R on the set {1, 2, 3} be defined by R = {(1, 2)}, then R is
   (A) Reflexive (B) transitive
   (C) Symmetric (D) none of these

7. Let A = {1, 2, 3} and consider the relationR = {1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1,3)}. Then R is
   (A) reflexive but not symmetric (B) reflexive but not transitive
   (C)symmetric and transitive (D) neither symmetric, nortransitive

8. If the set A contains 5 elements and the set B contains 6 elements, then thenumber of one-one and onto mappings from A to B is
   (B) 720 (B) 120
   (C) 0 (D) none of these

9. Which of the following functions from Z into Z are bijections?
   (D) $f(x) = x^3$ (B) $f(x) = x + 2$
   (E) $f(x) = 2x + 1$ (D) $f(x) = x^2 + 1$

10. Let $f : [2, \infty) \rightarrow R$ be the function defined by $f(x) = x^2 - 4x + 5$, then the range offis
   (F) R (B) $[1, \infty)$
   (G) $[4, \infty)$ (B) $[5, \infty)$

<u>Short Answar Type Questions:</u>

1. If n(A) = p and n(B) = q, then the number of relations from set A to set B = ________.

2. A function is called an onto function, if its range is equal to ________.

3. Find (gof)(x), if f(x) = |x|, g(x) = |5x + 1|.

4. Show that function f: N → N, given by f(x) = 2x, is one – one.

5. Let S = {1, 2, 3} Determine whether the function f: S → S defined as below haveinverse.

   f = {(1, 1), (2, 2), (3, 3)}

6. Let f : {1, 3, 4} → {1, 2, 5} and g : {1, 2, 5} → {1, 3} be given by f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down gof.

7. Consider f: {1, 2, 3} {a, b, c} given by f(1) = a, f(2) = b and f(3) = c find f^{-1} and showthat $(f^{-1})^{-1} = f$.

8. Is g = {(1, 1), (2, 3), (3, 5), (4, 7)} a function? If g is described by g(x) = ax + b, then what value should be assigned to a and b.

9. Let the function f : R → R be defined by f(x) = cosx, x ϵ R. Show that f is neither one-one nor onto.

10. Let L be the set of all lines in plane and R be the relation in L define if R = {(L_1, L_2): L_1Is perpendicularto L_2 } .Show that R is symmetric but neither reflexive nor transitive.

11. If the function f : R R is given by f (x) = x^2 + 2 and g: R → R is given bythen find fog and gof and hence find fog(2) and gof(- 3).

12. Show that f: R →R defined by f (x) = [x] is neither one-one nor onto.

13. Find fog for f(x) = e^x ; g(x) = log x

14. Show that the function f : R →R given by f(x) = 3x – 4 is a bijection.

15. Find f$\circ$g and g$\circ$f if f(x) = x^2 + 2 and g(x) = 1 - $\frac{1}{1-x}$

16. Let f:R$\to$R defined by f(x) = 2x – 3 and g : R$\to$R defined by g(x) = $\frac{x+3}{2}$. Show that f$\circ$g = I_R = g$\circ$f.

17. Show that the function f: R $\to$R given by f(x) = x^3 + x is a bijection.

Chapter-2
INVERSE TRIGONOMETRIC FUNCTIONS

KEY POINTS :

$\sin^{-1}$	:	$[-1, 1]$	$\rightarrow$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}$	:	$[-1, 1]$	$\rightarrow$	$[0, \pi]$
cosec^{-1}	:	$\mathbf{R} - (-1,1)$	$\rightarrow$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$\sec^{-1}$	:	$\mathbf{R} - (-1, 1)$	$\rightarrow$	$[0, \pi] - \{\frac{\pi}{2}\}$
$\tan^{-1}$	:	$\mathbf{R}$	$\rightarrow$	$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}$	:	$\mathbf{R}$	$\rightarrow$	$(0, \pi)$

5. (i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$, $xy < 1$

(ii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$, $xy > -1$

(iii) $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$, $|x| < 1$

6. (i) $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}$, $|x| \leq 1$

(ii) $2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}$, $x \geq 0$

(iii) $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$, $-1 < x < 1$

One mark questions:

Q.1 Find the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}(\frac{-1}{2})\right]$.

Q.2 If $\sin(\sin^{-1}\frac{1}{5} + \cos^{-1}x) = 1$. Find the value of x.

Q.3 Find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

Q.4 Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$

Q.5 Find the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

Q.6 Find the value of $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

Two marks questions:

Q.7 If $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, then find the value of *x*.

Q.8 Write the function in the simplest form:

$\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right),\ 0<x<\pi$

Q.9 Prove that :

$$\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$$

Q.10 Express in the simplest form

$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x\neq 0$

Q.11 Solve $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$.

Three marks questions:

Q.12 Solve $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x, (x>0)$

Q.13 Prove $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$, $x\in(0,1)$

Q.14 Prove that :

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\ \frac{\pi}{4}\right)$$

Q.15 Prove that : $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$, $-1/\sqrt{2}\le x\le 1$.

Chapter- 3
MATRICES

KEY POINTS:

(1) **Matrix**: A Matrix is a rectangular arrangement of number or functions arranged into a fixed number of rows and columns.

(2) **Order 0f Matrix**: The order of a matrix is defined by the number of rows and columns of that matrix.

(3) **Square Matrix**: A Matrix in which the no. of rows is equal to the no. of column's say n, is called a square matrix of order n.

(4) **Diagonal Matrix**: A square matrix $[a_{ij}]$ is said to be a diagonal matrix if $a_{ij} = 0$ for $i \neq j$.

(5) **Scalar Matrix**: A Square matrix $A = [a_{ij}]_{nxn}$ is called a sca;ar matrix of (i) $a_{ij} = 0$ for all $i \neq j$ and (ii) $a_{ij} = c \ \forall\ i = j$, where $c \neq 0$

(6) **Identity or Unit Matrix**: A Square Matrix $A = [a_{ij}]nxn$ is called an identity or a Unit matrix, if (i) $a_{ij} = 0$ for all $i \neq j$ (ii) $a_{ii} = 1 \ \forall\ i = j$.

(7) **Null or Zero Matrix**: A Matrix whose all elements are zero is called a null matrix or a zero matrix. ie.,$A = [a_{ij}]$ is null matrix if $a_{ij} = 0 \ \forall$ i,j

(8) **Equal Matrices**: Two matrices $A = [a_{ij}]_{mxn}$ and $B = [b_{ij}]_{mxn}$ of the same order are equal if $a_{ij} = b_{ij} \forall$ i, j

(9) If $A = [a_{ij}]_{mxn,}$ $B = [b_{ij}]_{mxn}$ of the same order are equal if
- (i) Commutativity: A + B = B + A
- (ii) Associativity: (A + B) + C = A + (B + C)
- (iii) Existence of Identity : A + 0 = 0 + A = A
- (iv) Existence of Invers: A + (-A) = (-A) + A = 0
- (v) Cancellation Laws: A + B = A + C →B =C and B + A = C + A → B =C

(10) If A and B are tswo matrices of the same order and k, l are scalars then
- (i) K(A + B) = kA + kB
- (ii) (k+ l)A = kA + lA
- (iii) (kl)A = k(lA) = l(kA)
- (iv) (-k)A = -(kA) = k(-A)
- (v) lA = A
- (vi) (-1)A = -A

(11) If A and B are two matrices of the same order then A – B = A + (-B)

(12) **Multiplication of Matrices**: Two matrices A and B are said to be defined for multiplication if the number of columns of first matrix is equal to the number of rows of second matrix. Let A $[aij]_{mxn}$ and $B = [b_{jk]nxp}$ then $AB = [C_{ij}]_{mxp}$ where
$C_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + ...+ a_{in}b_{nj} = \sum_{r=1}^{n} airbrj$

(13) Properties of matrix multiplication
- (i) Matrix multiplication is not commutative
- (ii) Matrix multiplication is associative
- (iii) Matrix multiplication is distributive over matrix addition.
- (iv) If A is an mxn matrix then $I_m A= A =AI_n$.

(v) If A is an mxn matrix $A_{mxn}O_{nxp} = O_{mxp}$ & $O_{pxm}A_{mxn} = O_{pxn}$

(vi) In a matrix multiplication the product of two non zero matrices may be a zero matrix

(14) **Transpose of a Matrix**: Let A = $[a_{ij}]$ be an mxn matrix then the transpose of A, denoted by A^Tor$\bar{A}$, is an nxm matrix such that $A^T = a_{ji}$

(15) Properties of Transpose of a Matrix:

(i) $(A^T)^T = A$ (ii) $(A+B)^T = A^T + B^T$

(ii) $(KA)^T = K\,A^T$ (iv) $(AB)^T = B^T A^T$

(iii) $(ABC)^T = C^TB^TA^T$

(16) **Symmetric Matrix**: A Square matrix A = $[a_{ij}]$ is called a symmetric matrix if $A = A^T$

(17) **Skew Symmetric Matrix** : A Square matrix A= $[a_{ij}]$ is called a skew symmetric matrix if $A^T = -A$

(18) Principal diagonal elements of askew symmetric matrix is zero.

(19) Every square matrix can be uniquely expressed as a sum of a symmetric and a skew-symmetric matrix.

(20) All positive integral powers of a symmetric matrix are symmetric.

(21) All odd positive integral powers of a skew-symmetric matrix are skew-symmetric.

Short And Long Answer Type Questions:

Q1-Form a 2× 3 matrix A whose elements are given by $a_{ij}=\frac{1}{2}|i-3j|$.

Q2-Find the values of a, b, c and d from the following equation:

$$\begin{bmatrix}2a+b & a-2b\\5c-d & 4c+3d\end{bmatrix} = \begin{bmatrix}4 & -3\\11 & 24\end{bmatrix}$$

Q3-. If A=$\begin{bmatrix}2 & 4\\2 & 3\end{bmatrix}$ and B=$\begin{bmatrix}3 & 1\\4 & 2\end{bmatrix}$, find AB.

Q4- If A=$\begin{bmatrix}2 & 4\\2 & 3\end{bmatrix}$, find $\bar{A}$.

Q5-. Define symmetric matrix.

Q6-.If X +Y =$\begin{bmatrix}7 & 0\\2 & 5\end{bmatrix}$and X – Y =$\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}$,then find X and Y.

Q7- If A =$\begin{bmatrix}7 & 0\\2 & 5\end{bmatrix}$, find $\frac{1}{2}$(A +$\bar{A}$).

Q8- Simplify cos A $\begin{bmatrix}cosA & sinA\\-sinA & cosA\end{bmatrix}$ + sin A $\begin{bmatrix}sinA & -cosA\\cosA & sinA\end{bmatrix}$.

Q9-If a matrix has 24 elements, what is the possible order it can have?

Q10- Find the values of x, y and z from the following equation:

$$\begin{bmatrix}x+y & 2\\5+z & xy\end{bmatrix} = \begin{bmatrix}6 & 2\\5 & 8\end{bmatrix}$$

Q11- If A=$\begin{bmatrix}2 & 4\\2 & 3\end{bmatrix}$ and B=$\begin{bmatrix}3 & 1\\4 & 2\end{bmatrix}$, find ah of following

(i) A + 3B (ii) A- 2B

Q12- If A = $\begin{bmatrix}1\\2\\3\end{bmatrix}$ $and\ B = \begin{bmatrix}2 & 3 & 4\end{bmatrix}$, then find AB.

Q13- If A=$\begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix}$, thenfind A^2-5A+6I .

Q14- If A = $\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}$, prove that A^3-6A^2+7A +2 I =0.

Q15- Define skew symmetric matrix.

Q16- Find the value of x and y, if $\begin{bmatrix}x+3y & y\\7-x & 4\end{bmatrix}$=$\begin{bmatrix}4 & -1\\0 & 4\end{bmatrix}$

Q17-If A =$\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$, Show that$A^2$ -5A+7I=O

Q18-For the matrix A=$\begin{bmatrix}1 & 5\\6 & 7\end{bmatrix}$, verify that

(i)(A + $\bar{A}$) is a symmetric matrix.

(ii)(A - $\bar{A}$) is a skew symmetric matrix.

Q19-Express matrix A=$\begin{bmatrix}3 & 5\\1 & -1\end{bmatrix}$ as the sum of symmetric and skew symmetric matrix.

Q20- Express matrix A= $\begin{bmatrix}3 & 3 & -1\\-2 & -2 & 1\\-4 & -5 & 2\end{bmatrix}$as the sum of symmetric and skew symmetric matrix.

<u>Multiple Choice Questions</u>

Q21- A=$[a_{ij}]_{m\times n}$ is a square matrix, if

(i) m< n (ii) m> n (iii) m= n (iv) none of these

Q22-The number of all possible matrices of order 3× 3 with each entry 0 or 1 is:

(i) 27 (ii) 18 (iii) 81 (iv 512

Q23- If A, B are symmetric matrices of same order, then (AB – BA) is a

(i) skew symmetric matrix (ii) symmetric matrix (iii) zero matrix (iv) Identity matrix

Q24- If A=$\begin{bmatrix}\cos\alpha & -\sin\alpha\\\sin\alpha & \cos\alpha\end{bmatrix}$, and A+ $\bar{A}$= I, then the value of α is

(i) $\frac{\pi}{6}$ (ii) $\frac{\pi}{3}$ (iii) π (iv) $\frac{3\pi}{2}$

Q25- If A=$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that A^2 = I, then

(i) 1+$\alpha^2+\beta\gamma$ =0 (ii) 1$-\alpha^2+\beta\gamma$ =0 (iii) 1$-\alpha^2-\beta\gamma$ =0 (iv)1+$\alpha^2-\beta\gamma$ =0

Q 26- If the matrix A is both symmetry and skew symmetric, then

(i) A is diagonal matrix(ii) A is zero matrix (iii) A is square matrix (iv) none of these

Q27 - If A is a square matrix such that A^2 = A, then $(\mathrm{I}+\ \mathrm{A})^3$ −7 A is equal to

(i)A (ii) I$-A$ (iii) I (iv) 3A

Q28- If A = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^2 is equal to

(i) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q29- If A= = $\begin{bmatrix} 0 & x+2 \\ 2x-3 & 0 \end{bmatrix}$ is skew symmetric matrix, then x is equal to

(i)$\frac{1}{3}$ (ii) 5 (iii) 3 (iv) 1

Q30- If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then (A+$\bar{A}$) is equal to

(i) $\begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$ (iv) none of these

Chapter-4

DETERMINANTS

KEY POINTS:

1. Only square matrices have determinants.

2.if A= $[a_{ij}]_{3\times3,}$ $then$ $|kA|$=$k^3|A|$.

3. If A is singular matrix, $|A|$=0

4.If A is skew symmetric matrix, $|A|$=0

5. If A is a square matrix of order n,$|$ $adj\ A|$=$|A|^{n-1}$

6. Area of triangle =$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

&. If A is a square matrix of order n, then A^{-1}= $\frac{adj\ A}{|A|}$ where$|A| \neq 0 i.e.$ A is non singular matrix.

1MARK QUESTIONS

1. Given that A is a square matrix of order 3x3 and |A|=-4. Find |adjA|
2. Let A= [a_{ij}] be a square matrix of order 3x3 and |A|= -7. Find the value of $a_{11}A_{21}$+$a_{12}A_{22}$+$a_{13}A_{23}$ Where A_{ij} is the cofactor of element a_{ij}.
3. Find adjointof A= $\begin{bmatrix} 2 & 4 \\ 1 & -3 \end{bmatrix}$
4. If A is a square matrix of order 3 an$d|A| = 6$ then find$|2A|$.
5. What is the value of$|3I_3|$?
6. Evaluate $\begin{vmatrix} 2cos\Theta & -2sin\Theta \\ sin\Theta & cos\Theta \end{vmatrix}$
7. If $A = \begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}$, Write the value of $|adj\ A|$.
8. If A is the square matrix of order 3 such that $|adj\ A$ | = 81, then find$|A|$.
9. If A is the square matrix of order 3, with $|A|$ = 9,then find $|2\ adj\ A|$.
10. If A is a singular matrix, then write $|A|$.

2 MARKS QUESTIONS

11. If A is a square matrix of order 3 such that A^2=2A, then find the value of |A|.
12. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that A^2 - 5A + 7I=0. Hence find A^{-1}.
13. If A=$\begin{bmatrix} 0 & 3 \\ -7 & 5 \end{bmatrix}$ then find k so that $kA^2 = 5A - 21I$.

14. Write the cofactors of the elements of second and third row $\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix}$.

3 MARKS QUESTIONS

15. Find the value of k, such that area of triangle with vertices (3, k), (2, 2), (-4, 1) is 3 square units.

16. .Given matrix A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, prove that $A^2 - 5A + 7I = 0$ and hence find A^{-1}.

17. If A = $\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, then verify that $(AB)^{-1} = B^{-1} A^{-1}$.

18. Find adjoint of matrix A, where A = $\begin{bmatrix} 2 & 1 & 4 \\ 0 & 3 & 2 \\ 1 & -1 & 1 \end{bmatrix}$

5MARKS QUESTIONS

19. Given A=$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B=$\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & -5 \end{bmatrix}$ Find the product of matrices AB and hence solve the system of equations x - y = 3,2x +3y + 4z = 17,y +2z =7

20. If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}, Using A^{-1} solve the system of equations:
2x – 3y + 5z = 11,3x + 2y – 4z = -5, x + y – 2z = -3

21. Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x - y + 2z = 1$, $2y - 3z = 1$ *and* $3x - 2y + 4z = 2$

22. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, Find A^{-1}. Hence solve the system of equations $x - 2y = 10, 2x - y - z = 8$ *and* $-2y + z = 7$.

Chapter 5

CONTINUITY AND DIFFERENTIABILITY

KEY POINTS:

1. If a function is **continuous** at x=a, then $\lim_{x\to a} f(x)$=f(a)

2. Learn all the basic formula of **derivatives .**The following table gives a list o**f derivatives of standard functions-**

F(x)	F'(x)
x^n	nx^{n-1}
consant	zero
sinx	cosx
cosx	-sinx
tanx	Sec^2x
secx	secx.tanx
cosecx	-cosecx.cotx
cotx	$-cosec^2x$
logx	$\frac{1}{x}$
e^x	e^x
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\cot^{-1} x$	$-\frac{1}{1+x^2}$
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$
$\text{cosec}^{-1} x$	$-\frac{1}{x\sqrt{x^2-1}}$
a^x	$a^x \log_e a$

The following rules were established as a part of algebra of dervatives :

(1) $(u \pm v)' = u' \pm v'$

(2) $(uv)' = uv' + vu'$

(3) $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$

For inverse trigonometric functions apply the following substitution :

(1) **For $\sqrt{x^2 - a^2}$ put x = acosecθ or a secθ**
(2) **For $\sqrt{x^2 + a^2}$ put x = atanθ or a cotθ**
(3) **For $\sqrt{a^2 - x^2}$ put x = asinθ or a cos θ**
(4) **For $\sqrt{\frac{x^2-a^2}{x^2+a^2}}$ put $x^2 = a^2\cos 2\theta$**
(5) **For $\sqrt{\frac{x-a}{x+a}}$ put x= acos2θ**

Ques1. For what value of a and b such that the function defined by

f(x) = $\begin{cases} 5, if\ x \leq 2 \\ ax + b, if\ 2 < x < 10 \\ 21,\ if\ 10 \leq x \end{cases}$ is a continuous function everywhere .

Ques2. Find the value of k so that the function f given by:

f(x) = $\begin{cases} \frac{k\cos x}{\pi - 2x}, if\ x \neq \frac{\pi}{2} \\ 3, if\ x = \frac{\pi}{2} \end{cases}$ is continuous at x = $\frac{\pi}{2}$.

Ques3. Find the value of k so that the function f given by:

f(x) = $\begin{cases} kx + 1, if\ x \leq 5 \\ 3x - 5, if\ x > 5 \end{cases}$ is continuous at x= 5.

Ques 4. For what value of k is the following function is continuous at x= 2

f(x) = $\begin{cases} 2x + 1 & if\ x < 2 \\ k & if\ x = 2 \\ 3x - 1 & if\ x > 2 \end{cases}$

Differentiate the functions with respect to x.

Ques 5. log$\sqrt{\frac{1-\cos x}{1+\cos x}}$

Ques 6. If y = log$\sqrt{\frac{1+\tan x}{1-\tan x}}$, prove that $\frac{dy}{dx} = sec2x$

Ques 7. sec(tan($\sqrt{x}$))

Ques 8. If y$\sqrt{x^2+1} = \log(\sqrt{x^2+1} - x)$ prove that $(x^2+1)\frac{dy}{dx}$ +xy+ 1= 0

Ques 9. Differentiate w.r.to x $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

Ques 10. Find $\frac{dy}{dx}$ if y= $\cos^{-1}(\frac{2x}{1+x^2})$

Ques 11.Differentiate w.r.to x $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$.

Ques 12. y = $\sec^{-1}(\frac{1}{4x^3-3x})$ then find $\frac{dy}{dx}$.

Ques 13. If $\sqrt{1-x^2}$ +$\sqrt{1-y^2}$ = a(x-y) then show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

LOGARITHMIC DIFFERENTIATION

Ques 14.If $x^y = e^{x-y}$ then prove that: $\frac{dy}{dx} = \frac{\log x}{\{\log(ex)\}^2}$

Ques 15. Find $\frac{dy}{dx}$ $, if\ y^x + x^y + x^x = a^b$

Ques 16. Differentiate w. r. to x $(\log x)^x + x^{\log x}$

Ques 17. Differentiate the function $x^{sinx} + sinx^{cosx}$ w. r. t. x

Ques 18.Find the derivative of y= $(\sin x)^{\cos x}$

Ques 19. Find $\frac{dy}{dx}$ if $x^y + y^x = a^b$.

Ques 20. Find $\frac{dy}{dx}$if y = $x^{(1+\frac{1}{x})} + (x+\frac{1}{x})^x$

DERIVATIVES OF FUNCTIONS IN PARAMETRIC FORMS

For parametric equations: x=f(θ) and y=g(θ)$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

Find$\frac{dy}{dx}$.Ques 21. X= $2at^2$, y = at^4

Ques 22. If $x = \sqrt{a^{\sin^{-1} t}}, y = \sqrt{a^{\cos^{-1} t}}$,show that $\frac{dy}{dx} = -\frac{y}{x}$

Ques 23. Find $\frac{dy}{dx}$,x = cosθ – cos2$\theta, y = sin\theta - sin2\theta$

SECOND ORDER DERIVATIVES

Ques 24. If y =5cosx – 3sinx, prove that $y_2 + y = 0$

Ques 25. If y = $(\tan^{-1} x)^2$, prove that $(1+x^2)^2 y_2 + 2x(1+x^2)y_1 = 2$.

Ques 26. If $y = Ae^{mx} + Be^{nx}$,show that $y_2 - (m+n)y_1 + mny = 0$

Chapter-6

APPLICATION OF DERIVATIVES

Key Points:

** Let I be an open interval contained in the domain of a real valued function f. Then f is said to be

(i) **increasing** on I if $x_1 < x_2$ in I $\Rightarrow$ f $(x_1) \le$ f (x_2) for all $x_1, x_2 \in$ I.

(ii) **strictly increasing** on I if $x_1 < x_2$ in I $\Rightarrow$ f $(x_1) <$ f (x_2) for all $x_1, x_2 \in$ I.

(iii) **decreasing** on I if $x_1 < x_2$ in I $\Rightarrow$ f $(x_1) \ge$ f (x_2) for all $x_1, x_2 \in$ I.

(iv) **strictly decreasing** on I if $x_1 < x_2$ in I $\Rightarrow$ f $(x_1) >$ f (x_2) for all $x_1, x_2 \in$ I.

** (i) f is **strictly increasing** in (a, b) if **$f'(x) > 0$** for each $x \in$ (a, b)

(ii) f is **strictly decreasing** in (a, b) **if $f'(x) < 0$** for each $x \in$ (a, b)

** Slope of the tangent to the curve y = f (x) at the point (x_0, y_0) is given by $\left[\frac{dy}{dx}\right]_{(x_0, y_0)}$ $(= f'(x_0))$.

** The **equation of the tangent** at (x_0, y_0) to the curve y = f (x) is given by $y - y_0 = f'(x_0)(x - x_0)$.

** **Slope of the normal** to the curve y = f (x) at (x_0, y_0) is $-\frac{1}{f'(x_0)}$.

** The **equation of the normal** at (x_0, y_0) to the curve y = f (x) is given by $y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$.

** If slope of the tangent line is zero, then tan θ = 0 and so θ = 0 which means the tangent line is parallel to the x-axis. In this case, the equation of the tangent at the point (x0, y0) is given by $y = y_0$.

** If $\theta \to \frac{\pi}{2}$, then $\tan\theta \to \infty$, which means the tangent line is perpendicular to the x-axis, i.e., parallel to they-axis. In this case, the equation of the tangent at (x_0, y_0) is given by $x = x_0$.

** Let f be a function defined on an interval I. Then

(a) f is said to have a maximum value in I, if there exists a point c in I such that $f(c) \ge f(x)$, for all $x \in I$.

The number f (c) is called the maximum value of f in I and the point c is called a point of maximum value of f in I.

(b) f is said to have a minimum value in I, if there exists a point c in I such that $f(c) \le f(x)$, for all $x \in I$.

The number f (c), in this case, is called the minimum value of f in I and the point c, in this case, is called a point of minimum value of f in I.

(c) f is said to have an extreme value in I if there exists a point c in I such that f (c) is either a maximum value or a minimum value of f in I.

The number f (c), in this case, is called an extreme value of f in I and the point c is called an extreme point.

* * **Absolute maxima and minima**

Let f be a function defined on the interval I and $c \in I$. Thenss

(a) f(c) is absolute minimum if $f(x) \geq f(c)$ for all $x \in I$.

(b) f(c) is absolute maximum if $f(x) \leq f(c)$ for all $x \in I$.

(c) $c \in I$ is called the critical point of f if $f'(c) = 0$

(d) Absolute maximum or minimum value of a continuous function f on [a, b] occurs at a or b or at critical points off.

(i.e. at the points where $f'$is zero)

If $c_1, c_2, \ldots, c_n$ are the critical points lying in [a , b], then absolute maximum value of f = $\max\{f(a), f(c_1), f(c_2), \ldots, f(c_n), f(b)\}$and absolute minimum value of f = $\min\{f(a), f(c_1), f(c_2), \ldots, f(c_n), f(b)\}$.

**** Local maxima and minima**

(a) A function f is said to have a local maxima or simply maximum value at x =a if $f(a \pm h) \leq f(a)$.

(b)A function f is said to have a local minima or simply a minimum value at x = a if $f(a \pm h) \geq f(a)$.

**** First derivative test** :A function f has a maximum at a point x = a if

(i) $f'(a) = 0$, and

(ii) $f'(x)$ changes sign from + ve to –ve in the neighborhood of 'a' (points taken from left to right).

However, f has a minimum at x = a, if

(i) $f'(a) = 0$, and

(ii) $f'(x)$ changes sign from –ve to +ve in the neighborhood of 'a'.

If $f'(a) = 0$ and f'(x) does not change sign, then f(x) has neither maximum nor minimum and the point 'a' is called point of inflexion.

The points where $f'(x) = 0$ are called stationary or critical points. The stationary points at which the function attains either maximum or minimum values are called extreme points.

Second derivative test

(i) a function has a maxima at x a if $f'(x)= 0$ and $f''(a) <0$

(ii) a function has a minima at x = a if $f'(x) = 0$ and $f''(a) > 0$.

Increasing & decreasing functions

LEVEL I

1. Show that $f(x) = x^3 - 6x^2 + 18x + 5$ is an increasing function for all $x \in R$.
2. Show that the function $x^2 - x + 1$ is neither increasing nor decreasing on (0,1)
3. Find the intervals in which the function $f(x) = \sin x - \cos x$, $0 < x < 2\pi$ isincreasing or decreasing.
4.Find the least value of 'a' such that the function x^2+ax+1 is increasing on [1,2].
5.Find the intervals in which $f(x) = -x^2 - 2x + 15$ is increasing or decreasing.
6. Indicate the interval in which the function $f(x) = \cos x$, $0 \le x \le 2\pi$ is decreasing.

7.Show that the function $f(x) = \dfrac{\sin x}{x}$ is strictly decreasing on $(0, \pi/2)$

8. Find the intervals in which the function $f(x) = \dfrac{\log x}{x}$ increasing or decreasing.

9.Find the interval in which the function $f(x)=2x^3 + 9x^2 + 12x + 20$ is (i) increasing (ii) decreasing
10. Find the interval in which the function $f(x) = (x+1)^3(x-1)^3$

LEVEL II

*1. Find the interval in which the function $f(x)=2x^3 - 9x^2 + 12x + 15$ is (i) increasing (ii) decreasing

(CBSE2010)
*2. Find the interval in which the function $f(x) = (x+1)^3(x-1)^3$
(CBSE2011)
*3. Find the intervals in which $f(x) = \log(1+x) - \dfrac{2x}{2+x}$ is increasing or decreasing.

(CBSE2012)
*4. Find the interval in which the function $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$ is increasing or decreasing

(CBSE2012)
*5. Separate the interval $[0, \pi/2]$ into sub intervals in which $f(x) = \sin^4 x + \cos^4 x$ is increasing or
decreasing

LEVEL III

1. Find the interval of monotonocity of the function $f(x) = 2x^2 - \log x$, $x \ne 0$
2. Prove that the function $y = \dfrac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ in $[0, \pi/2]$

[CBSE 2011]

3.Find the intervals in which f(x) = $\log(1+x) - \frac{2x}{2+x}$ is increasing or decreasing.

4.Find the interval in which the function f (x) =sinx +cosx, $0 \le x \le 2\pi$ is increasing or decreasing

5.Separate the interval [0, $\pi/2$] into sub intervals in which f(x) = $\sin^4 x + \cos^4 x$ is increasing or decreasing .

Tangents &Normals

LEVEL-I

1.Find the equations of the normals to the curve $3x^2 - y^2 = 8$ which are parallel to the line $x + 3y = 4$.

2. Find the point on the curve $y = x^2$ where the slope of the tangent is equal to the x-coordinate of
the point.

3. At what points on the circle $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangent is parallel to x axis ?

4. Find the equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3)

5. For the curve $y = 2x^2 + 3x + 18$, find all the points at which the tangent passes through the origin.

6. Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4= 0$

7. Show that the equation of tangent at (x_1 , y_1) to the parabola $yy_1=2a(x + x_1)$.

[CBSE 2012Comptt.]

LEVEL- II

*Que1. . Find the equations of all lines having slope 0, which are tangent to the curve
$y = \frac{1}{x^2 - 2x + 3}$

***Que2. for the curve $y = 4x^3 - 2x^5$, find all the points at which the tangents passes through the origin.

(CBSE2013)

*Que3. Prove that the curves $x = y^2$ and $xy = k$ intersect at right angles if $8k^2 = 1$.
(CBSE 2008)

*Que4. Find the equation of tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0,y_0)

*Que5. Find the equation of normal to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x+14y +4 = 0$.

(CBSE2010)

LEVEL-III

1. Find the equation of the tangent line to the curve y = $\sqrt{5x-3}$ –2 which is parallel to the line
4x –2y +3 =0
2. Show that the curve $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 - 2y = 0$ cut orthogonally at the point (0,0)
3. Find the condition for the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $xy = c^2$ to intersect orthogonally.
4. Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta$, $y = a\sin\theta - a\theta\cos\theta$
is at a constant distance from the origin.

[CBSE 2013]

Maxima & Minima

LEVEL I

1. Find the maximum and minimum value of the function $f(x) = 3 - 2\sin x$
2. Show that the function $f(x) = x^3 + x^2 + x + 1$ has neither a maximum value nor a minimum value
3. Find two positive numbers whose sum is 24 and whose product is maximum
4. Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \cos^2 x + \sin x$, $x \in [0, \pi]$.
5. Prove that the area of a right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.
6. A piece of wire 28(units) long is cut into two pieces. One piece is bent into the shape of a circle and other into the shape of a square. How should the wire be cut so that the combined area of the two figures is as small as possible.
7. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

LEVEL II

*Que1. A window is in the form of rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum sunlight through the whole opening. Explain the importance of sunlight. (CBSE2011)

*Que.2. Que2. Show that the height of cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3.}}$. (CBSE 2012,2013)

*Que3. Show that the semi-vertical angle of a cone of maximum volume and of given slant height is

$\tan^{-1}\sqrt{2}$.

(CBSE 2008,2011)

*Que4. Find the maximum and minimum values of the function f(x)= $|x+3|$ for all $x \in R$.

*Que5. Length of three sides of a trapezium other than base is equal to 10cm each, then find the area of the trapezium when it is maximum? (CBSE 2010,2013)

*Que6. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth in 2m and volume is 8 m^3. If building of tank costs Rs.70 per sq. metre for the base and Rs.45 per sq.metre for sides, what is the cost of least expensive tank? (CBSE 2009)

*Que7. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(CBSE 2013)

*Que8. Find the point on the curve $y^2= 2x$ which is at minimum distance from the point (1,4)

(CBSE 2009,2011)

LEVEL III

1.Find the area of the greatest isosceles triangle that can be inscribed in a given ellipse having its vertex coincident with one extremity of major axis.

2.An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

[CBSE 2012 Comptt.]

3.A window is in the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

[CBSE 2011]

4. Show that a cylinder of a given volume which is open at the top has minimum total surface area, when its height is equal to the radius of its base.

5. Tangent to the circle $x^2 + y^2= 4$ at any point on it in the first quadrant makes intercepts OA and OB on x and y axes respectively, O being the centre of the circle. Find the minimum value of (OA + OB). **[CBSE 2015]**

<u>Questions for self evaluation</u>

1.3. Find the intervals in which the following function is strictly increasing or decreasing:

$f(x) = -2x^3 - 9x^2 - 12x + 1$

4. Find the intervals in which the following function is strictly increasing or decreasing: $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$

5. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

6. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is
(a) parallel to the line $2x - y + 9 = 0$ (b) perpendicular to the line $5y - 15x = 13$.

7. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

9. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. **[CBSE 2013,2014]**

10. An open topped box is to be constructed by removing equal squares from each corner of a 3metre by 8metre rectangular sheet of aluminum and folding up the sides. Find the volume of the largest such box.

Chapter-7

INTEGRALS

Key Points:

1. $\int x^n dx = \frac{x^{n+1}}{n+1}$ **+c**

2. $\int e^x dx = e^x$ **+c**

3. $\int a^x dx = \frac{a^x}{\log x}$ **+c**

4. $\int \frac{1}{x} dx = \log x + c$

5. $\int \sin x \, dx = -\cos x$ **+c**

6. $\int \cos x \; dx = \sin x$ **+c**

7. $\int \tan x \, dx = -\log|\cos x|$**+c or** $\log|\sec x|$**+c**

8. $\int \cot x \, dx = \log|\sin x|$**+c**

9. $\int \sec x \, dx = \log|\sec x + tanx|$**+c**

10. $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - cotx|$**+c**

11. $\int sec^2$**x dx=** $\tan x$**+c**

12. $\int cosec^2$**x dx=** $-\cot x$**+c**

13. $\int secx \; tan \; x$**dx=sec x +c**

14. $\int cosecx \; cot \; x$**dx=**$-$**cosec x +c**

INDEFINITE INTEGRALS

1. $\int e^{-2\log\cos x} dx$ Ans. tanx +c

2. $\int \sqrt{1+2\tan x(\tan x+\sec x)}dx$ Ans. log |secx(secx+tanx)|+c

3. $\int \frac{\sin(\tan^{-1} x)}{1+x^2}dx$ Ans. –cos($\tan^{-1}$x)+c

4. $\int \frac{x^2}{1+x^3}dx$ Ans. 1/3 log |1+x^3|+c

5. $\int \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}}\,dx$ Ans.1/3 $(\sin^{-1}x)^3$ +c

6. $\int \frac{1}{x(x^n+1)}dx$ Ans. $\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + c$

7. $\int \frac{2x+1}{(x+x^2)}\,dx$ Ans. $\boldsymbol{log|x+x^2|+c}$

8. $\int \frac{1}{x^2+4x+8}\,dx$ Ans. $\frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right)+c$

9. $\int \frac{dx}{x+\sqrt{x}}$ Ans. $\boldsymbol{log|1+\sqrt{x}|+c}$

10. $\int \frac{Sec2(\log x)}{x}\,dx$ Ans. $\frac{1}{2}\boldsymbol{log}\left|\boldsymbol{sec(2logx)++tan(2logx)}\right|+\boldsymbol{c}$

11. $\int \frac{1+\sin 3x}{3x-\cos 3x}\,dx$ Ans. $\frac{1}{3}\boldsymbol{log|3x-cos3x|+c}$

12. $\int \frac{sinx}{\sin(x-a)}\,dx$ Ans. $xcosa + sinalog|\sin(x-a)|+c$

13. $\int e^x[\sec x + \log(\sec x + \tan x)]dx$ Ans. $\boldsymbol{e^x log|secx+tanx|+c}$

14. $\int \frac{1}{\sin^2 x\cos^2 x}dx$ Ans.$\boldsymbol{-2cot2x+c}$

15. $\int \frac{\sin 2x - 2\cos x}{5-\cos^2 x - 2\sin x}\,dx$ Ans.log|5-cos^{2}x-2sinx|+c

16. $\int e^{3\log x}(x^4)dx$ Ans. x^8/8+c

17. $\int Co\sec^2 x.Sec^2 x.dx$ Ans. -2cot2x+c

18. $\int \frac{\cos x + \sin x}{\cos x - \sin x}\,dx$ Ans. –log|cosx-sins|+c

19. $\int \sin^{-1}\cos x\,dx$ Ans. $\frac{\pi}{2}\boldsymbol{x} - \frac{x^2}{2}$+C

20. $\int\{f''(ax+b)/f(ax+b)\}\,dx$ Ans. 1/a log |f(ax+b)|+c

21. $\int\left(x^a + a^x + e^x.a^x + \sin a\right)dx$ Ans. $\frac{a^{a+1}}{a+1} + \frac{a^x}{loga} + \frac{(ea)^x}{\log(ea)} + xsina + c$

22. $\int x^2 e^{x^3}dx$ Ans. $\frac{1}{3}e^{x^3} + c$

23. $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$ Ans. $5\sqrt{x^2+4x+10} - 7log\left|(x+2) + \sqrt{x^2+4x+10}\right| + c$

24. Prove that $\int_0^a f(x)dx = \int_o^a f(a-x)dx$

25.Evaluate : $\int \frac{x^2+1}{x^4+1}$dx Ans. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + c$

26. $\int \frac{e^x}{\sqrt{5-4e^x-e^{2x}}}$dx Ans. $\sin^{-1}\left(\frac{e^x+2}{3}\right) + c$

27. $\int \frac{1}{(x+1)(x+2)}dx$ **Ans.** $log\left|\frac{x+1}{x+2}\right| + c$

28. $\int \frac{x-\sin x}{1-\cos x}dx$ **Ans. -xcotx/2+c**

29. $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}$ **dx .** **Ans.** $\frac{2(2x-1)}{\pi}\sin^{-1}\sqrt{x} + \frac{2\sqrt{x-x^2}}{\pi} - x + c$

30. $\int \frac{3x-2}{(x+1)^2(x+3)}dx.$ **Ans.** $\frac{11}{4}log\left|\frac{x+1}{x+3}\right| + \frac{5}{2(x+1)} + c$

31

$\int Cos^4 x dx$ Ans. $\frac{1}{8}\left\{3x - 2sin2x + \frac{sin4x}{4}\right\} + c$

32.

$\int \frac{1}{\cos(x-a)\cos(x-b)}dx$ Ans. $\frac{1}{\sin(a-b)}log\left|\frac{\cos(x-a)}{\cos(x-b)}\right| + c$

33.

$\int \sqrt{\cot x}\,dx$ Ans. $\frac{-1}{\sqrt{2}}\tan^{-1}\left(\frac{cotx-1}{\sqrt{2cotx}}\right) - \frac{1}{2\sqrt{2}}log\left|\frac{cotx+1-\sqrt{2cotx}}{cotx+1+\sqrt{2cotx}}\right| + c$

34.

$\int \frac{x^2-1}{x^4+1}\ dx$ Ans. $\frac{1}{2\sqrt{2}}log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right| + c$

35.

$\int \frac{x^2+4}{x^4+x^2+16}\,dx$ Ans.$\frac{1}{3}\tan^{-1}\left(\frac{x-\frac{4}{x}}{3}\right)+c$

36

$\int \frac{2+\sin x}{1+\cos x}e^{\frac{x}{2}}\,dx$ Ans.$2e^{x}tanx/2+c$

37.

$\int \frac{e^{x}}{6+5e^{x}-e^{2x}}\,dx$ Ans.$\log\left|\frac{e^{x}-2}{3-e^{x}}\right|+c$

38.

$\int \frac{dx}{\sqrt{7-6x-x^2}}$ Ans.$\sin^{-1}\left(\frac{x+3}{2}\right)+c$

39.

$\int \frac{dx}{e^{x}-1}$ Ans. $log|1-e^{-x}|$+c

$\int \frac{dx}{\sqrt{6+x-x^2}}$ Ans. $\sin^{-1}\left(\frac{2x-1}{5}\right)+c$

40.

41. $\int 5^{5^{5^x}}.5^{5^x}.5^x\,dx$ Ans. $5^{5^{5^x}}\frac{1}{(log5)^3}+c$

42. $\int\sqrt{e^x-1}dx$ Ans. $2\left[\frac{e^x-1}{2}-\sqrt{e^x-1}+log(1+\sqrt{e^x-1})\right]+c$

43. $\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$ Ans.$5\sqrt{x^2+4x+10}-7log\left|x+2+\sqrt{x^2+4x+10}\right|$+c

44. $\int \frac{(x^2+1)e^{x}}{(x+1)^2}\,dx$ Ans.$\frac{x-1}{x+1}e^{x}+c$

$\int \frac{(x-3)e^{x}}{(x-1)^3}dx$ Ans.$\frac{e^x}{2}$+c

45.

46. $\int \frac{x+3}{\sqrt{5-4x+x^2}}$ Ans.$-\sqrt{5-4x-x^2}+\sin^{-1}\left(\frac{x+2}{3}\right)+c$

47. $\int \frac{e^{2x}-1}{e^{2x}+1}dx$ Ans. $\log|e^x+e^{-x}|+c$

48. $\int \frac{(3\sin\theta-2)\cos\theta}{5-\cos^2\theta-4\sin\theta}d\theta$ Ans. $3\log|2-\sin\theta|+\frac{4}{2-\sin\theta}+c$

49. $\int \frac{dx}{1+\sin x}$ Ans. $\tan x-\sec x+c$

50. $\int \frac{1}{x^{1/2}+x^{1/3}}dx$ Ans.$2\sqrt{x}-3x^{1/3}+6x^{1/6}-6\log\left(1+x^{\frac{1}{6}}\right)+c$

51. $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx$ Ans. 2(sinx+xcosα) $+c$

52. $\int \frac{1}{\log x}-\frac{1}{(\log x)^2}\ dx$ Ans.$\frac{x}{\log x}+c$

53. $\int [\log(\log x)+\frac{1}{(\log x)^2}]dx$ Ans.$x\log(\log x)-\frac{x}{\log x}+c$

54. $\int \frac{x^2}{(x^2+1)(x^2+4)}\ dx$ Ans. -1/3tan^{-1}x +2/3tan^{-1}(x/2)+c

55. $\int \frac{\sqrt{\tan x}}{\sin x.\cos x}dx$ Ans.$2\sqrt{\tan x}+c$

56. $\int\left(\sqrt{\cot x}+\sqrt{\tan x}\right)dx$ Ans.$\sqrt{2}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)$+c

57. $\int \frac{\sqrt{x^2+1}}{x^4}\left(\log\left(x^2+1\right)-2\log x\right)dx$ Ans.$-1/3\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)-2/3\right]+c$

58. $\int \frac{x^2}{(x^2+1)(x^2+4)}dx$ Ans. $-\frac{1}{3}\tan^{-1}x+\frac{2}{3}\tan^{-1}\frac{x}{2}+c$

60. $\int \frac{\sin 2x}{(\sin x+1)(\sin x+2)}dx$ Ans. $\log\left|\frac{(2+\sin x)^4}{(1+\sin x)^2}\right|+c$

DEFINITE INTEGRAL

1. $\int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}\,dx$ Ans. $\frac{\pi^2}{16}$

2. $\int_0^{\pi} \frac{x\tan x}{\sec x\,\mathrm{cosec}\,x}dx$ Ans. $\frac{\pi^2}{4}$

3. $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x}dx$ Ans. 1/40log9

4. $\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)dx$ Ans. $\frac{\pi}{2}\log(\frac{1}{2})$

.5. $\int_0^{3/2}|x\cos\pi x|\,dx$ Ans. $\frac{5}{\pi}-\frac{1}{\pi^2}$

6. $\int_0^2|x^2+2x-3|\,dx$ Ans. 4

7.Prove that: $\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx = a\pi$

8. $\int_0^{\frac{\pi}{4}}\log(1+\tan x)dx$ Ans. $\frac{\pi}{8}\log 2$

9. $\int_0^{\pi/2}\frac{1}{1+\sqrt{\cot x}}dx$ Ans. $\frac{\pi}{4}$

10. $\int_{-1}^{2}|x^3-x|dx$ Ans. 11/4

11. $\int_0^{\pi/2}\frac{\cos x}{(1+\sin x)(2+\sin x)}dx$ Ans. log4/3

12. $\int_1^4[|x-1|+|x-2|+|x-3|]dx$ Ans.19/2

13. $\int_0^{\pi/2}\frac{\cos x}{1+\sin^2 x}dx$ Ans. $\frac{\pi}{4}$

14. $\int_0^{\pi/2}\frac{dx}{1+\tan x}$ Ans. $\frac{\pi}{4}$

15. $\int_0^{\pi}\frac{x\,dx}{a^2\cos^2 x+b^2\sin^2 x}$ Ans. $\frac{\pi^2}{2ab}$

16. $\int_{0}^{a} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$ Ans. a/2

17. $\int_{1}^{3} \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx$ Ans. 1

18. $\int_{0}^{\pi} \frac{x}{1+\sin x}dx$ Ans. π

19. $\int_0^1 \cot^{-1}(1-x+x^2)dx$ Ans. $\frac{\pi}{2} - \log 2$

20. $\int_{0}^{\pi} \log(1+\cos x)\,dx$ Ans. $-\pi\log 2$

21. Evaluate: $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$ Ans. $\frac{\pi^2}{4}$

22. $\int_{-1}^{3/2} |x \, Sin\pi \, x| \, dx$ Ans. $\frac{1+3\pi}{\pi^2}$

23. Evaluate $\int_{-1}^{1} (|x-1|+|x|+|x+1|)dx$ Ans.5

24. $\int_{-1}^{2} \frac{|x|}{x}dx$ Ans.1

Questions for Practice

1. INDEFINITE INTEGRALS:

2. EVALUATE : $\int \frac{X+3}{x^2+4X+3}dx$. [2006]
3. EVALUATE : $\int \frac{1}{1-\sin x}dx$. [2002]
4. EVALUATE : $\int \frac{1+\sin x}{1-\sin x}dx$. [2000]
5. EVALUATE : $\int \frac{dx}{\sqrt{x+1}+\sqrt{x+2}}$. [2002]
6. EVALUATE : $\int \tan^{-1}\sqrt{\frac{1-\cos 2x}{1-\cos 2x}}dx$. [2003]
7. EVALUATE : $\int \tan^{-1}\sqrt{\frac{1-\sin x}{1-\sin x}}dx$. [2003 ,2006]
8. EVALUATE : $\int \frac{dx}{\sqrt{x+3}-\sqrt{x+2}}$. [2002]
9. EVALUATE :. $\int (1-x)\sqrt{x}\,dx$ [2012]
10. EVALUATE : $\int (3\cot x - 2\tan x)^2\,dx$ [2001]
11. EVALUATE : $\int sec^2(7-4x)dx$ [2010]
12. EVALUATE : $\int \sin 7x \sin x \, dx$[2001]
13. EVALUATE : $\int \cos 2x \cos 4x \, dx$[2007]
14. EVALUATE : $\int (2)^x\,dx$ [2010]
15. EVALUATE:$\int sin^4 x\,dx$ [2004]
16. EVALUATE : $\int \frac{2\cos x}{3sin^2 x}dx$ [2011]
17. EVALUATE : $\int \frac{\sin x}{\sin(x-a)}dx$ [2005]
18. EVALUATE : $\int \sin 7x \sin 5x \, dx$[2000]

19. EVALUATE : $\int \cos x \cos 7x\, dx$[2002]
20. EVALUATE : $\int \cos 4x \cos 3x\, dx$ [2007]
21. EVALUATE : $\int \frac{dx}{50+2x^2}$. [2002]
22. EVALUATE : $\int \frac{dx}{32-2x^2}$. [2007]
23. EVALUATE : $\int \frac{dx}{\sqrt{1-x^2}}$. [2011]
24. EVALUATE : $\int \sin 4x \cos 7x\, dx$ [2007]
25. EVALUATE : $\int \sin x \sin 2x \sin 3x\, dx$ [2012]
26. EVALUATE : $\int \frac{1+\cos x}{1-\cos x} dx$. [2000]
27. EVALUATE : $\int \frac{1+\sin 2x}{x+sin^2 x} dx$. [2000]
28. EVALUATE : $\int \frac{1-\sin 2x}{x+cos^2 x} dx$. [2000]
29. EVALUATE : $\int \frac{1+\tan x}{x+\log \sec x} dx$. [2000]
30. EVALUATE : $\int \frac{1+\cot x}{x+\log \sin x} dx$ [2000]
31. EVALUATE : $\int \frac{x}{e^{x^2}} dx$
32. EVALUATE : $\int \frac{e^{2x}-e^{2x}}{e^{2x}+e^{2x}} dx$
33. EVALUATE : $\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)} dx$ [2006]
34. EVALUATE : $\int \frac{1}{\cos(x-a)\cos(x-b)} dx$
35. EVALUATE : $\int \frac{sec^2 x}{3+\tan x} dx$ [2009]
36. EVALUATE : $\int \frac{sec^2(\log x)}{x} dx$ [2001]
37. EVALUATE : $\int \frac{(\log x)^2}{x} dx$ [2009]
38. EVALUATE : $\int \frac{cosec^2(\log x)}{x} dx$ [2000]
39. EVALUATE : $\int \frac{1}{x+x\log x} dx$
40. EVALUATE : $\int \frac{(x+1)(x+\log x)^2}{x} dx$ [2002]
41. EVALUATE : $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$
42. EVALUATE : $\int \frac{\sin(2\tan^{-1} x)}{1+x^2} dx$ [2002]
43. EVALUATE : $\int \frac{sec^2(2\tan^{-1} x)}{1+x^2} dx$
44. EVALUATE : $\int tan^3 x dx$ [2004]
45. EVALUATE : $\int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x dx$
46. EVALUATE : $\int \frac{(x^4-x)^{\frac{1}{4}}}{x^5} dx$ [2006]
47. EVALUATE : $\int (4x+2)\sqrt{x^2+x+1} dx$
48. EVALUATE : $\int \frac{\log x}{x} dx$ [2010]
49. EVALUATE : $\int \frac{2x.\tan^{-1}(x^2)}{1+x^4} dx$ [2006]
50. EVALUATE : $\int \frac{\sqrt{16+(\log x)^2}}{x} dx$ [2005]
51. EVALUATE : $\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)} dx$ [2006]
52. EVALUATE : $\int (2x+1)\sqrt{x^2+x+1}\, dx$ [2007]
53. EVALUATE : $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$ [2009]

54. EVALUATE : $\int \frac{x^2}{1+x^3} dx$ [2008]
55. EVALUATE : $\int \frac{1+\cot x}{x+\log \sin x} dx$ [2000]
56. EVALUATE : $\int \frac{1+\sin 2x}{x+ sin^2 x} dx$ [2000]
57. EVALUATE : $\int \frac{\log(\sin x)}{\tan x} dx$ [2010]
58. EVALUATE : $\int \frac{sec^2 x}{cosec^2 x} dx$ [2011]
59. EVALUATE : $\int \frac{(1+\log x)^2}{x} dx$ [2009]
60. EVALUATE : $\int \frac{\sin 2x}{a^2 sin^2 x+b^2 cos^2 x} dx$ [2005]
61. EVALUATE : $\int \frac{1}{5\cos x-12\sin x} dx$ [2005
62. EVALUATE : $\int \frac{\cos(x+a)}{\sin(x+b)} dx$ [2006]
63. EVALUATE : $\int \frac{x^2}{(1+x^{3)(2+x^3)}} dx$ [2003]
64. EVALUATE : $\int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx$ [2007]
65. EVALUATE : $\int \frac{1}{x(1+x^n)} dx$
66. EVALUATE : $\int \frac{\sin x}{(2+\cos x)(5+\cos x)} dx$ [2003]
67. EVALUATE : $\int \frac{\sin 2x}{(1-\sin x)(2-\cos 2x)} dx$
68. EVALUATE : $\int \frac{1}{e^x-1} dx$
69. EVALUATE : $\int \frac{2x-3}{(x^2-1)(2x+3)} dx$ [2003]
70. EVALUATE : $\int \frac{2x}{(2+x^2)(x^2+3)} dx$ [2003]
71. EVALUATE : $\int \frac{2}{(1-x)(1+x^2)} dx$ [2012]
72. EVALUATE : $\int \frac{x}{(x^2+1)(x+1)} dx$ [2002]
73. EVALUATE : $\int \frac{\tan^{-1} x}{(1+x^2)^2} dx$ [2003]
74. EVALUATE : $\int \frac{1}{x^2-6x+8} dx$ [2002]
75. EVALUATE : $\int \frac{1}{x^2-7x+10} dx$[2000]
76. EVALUATE : $\int \frac{x+3}{x^2+4x+3} dx$ [2006]
77. EVALUATE : $\int \frac{1+x^2}{1+x^4} dx$ [2006,2007]
78. EVALUATE : $\int \frac{1}{x^2+8x+20} dx$ [2002]
79. EVALUATE : $\int \frac{1}{x^2-4x+8} dx$ [2002]
80. EVALUATE : $\int \frac{x}{x^4-x^2+1} dx$ [2003]
81. EVALUATE : $\int \frac{x+3}{x^2-2x-5} dx$
82. EVALUATE : $\int \frac{2x+1}{2x^2+4x-3} dx$ [2006]
83. EVALUATE : $\int \frac{2\sin x+3\cos x}{3\sin x+4\cos x} dx$ [2006]
84. EVALUATE : $\int \frac{3x+1}{2x^2-2x+3} dx$ [2006]
85. EVALUATE : $\int \sqrt{\tan x}\, dx$
86. EVALUATE : $\int \frac{1-x^2}{1+x^4} dx$ [2007]
87. EVALUATE : $\int \frac{1}{\sqrt{7-6x-x^2}} dx$ [2002]

88. EVALUATE : $\int \frac{1}{\sqrt{4-2x-2x^2}} dx$ [2003]
89. EVALUATE : $\int \frac{dx}{\sqrt{16-2x-2x^2}}$ [2003]
90. EVALUATE : $\int \frac{dx}{\sqrt{8-4x-2x^2}}$ [2003]
91. EVALUATE : $\int \frac{dx}{\sqrt{15-8x^2}}$ [2002]
92. EVALUATE : $\int \frac{e^x dx}{\sqrt{5-4e^x-e^{2x}}}$ [2009 ,03,05]
93. EVALUATE : $\int \frac{dx}{\sqrt{(x-1)(x-2)}}$ [2006]
94. EVALUATE : $\int \frac{x+3}{\sqrt{5-4x-x^2}} dx$
95. EVALUATE : $\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx$ [2011]
96. EVALUATE : $\int \frac{2x+5}{\sqrt{x^2+3x+2}} dx$ [2001]
97. EVALUATE : $\int \frac{2x+3}{\sqrt{x^2+4x+3}} dx$ [2000]
98. EVALUATE : $\int \frac{4x+1}{\sqrt{2x^2+x-3}} dx$ [2001]
99. EVALUATE : $\int \frac{2x+3}{\sqrt{2x^2+x+1}} dx$ [2002]
100. EVALUATE : $\int \sqrt{x^2+4x+6}dx$
101. EVALUATE : $\int \sqrt{x^2+6x-4}dx$
102. EVALUATE : $\int \frac{1}{\sqrt{x^2-3x+2}} dx$ [2006]
103. EVALUATE : $\int \frac{1}{\sqrt{8+2qx-x^2}} dx$ [2002]
104. EVALUATE : $\int \frac{\sin x}{\sqrt{sin^2 x-2\sin x-3}} dx$ [2002]
105. EVALUATE : $\int \frac{1}{x^2+8x+20} dx$ [2002]
106. EVALUATE : $\int (1+x)\log x\, dx$ [2002]
107. EVALUATE : $\int \log(2+x^2)\, dx$ [2001]
108. EVALUATE : $\int \frac{\sin^{-1} x}{x^2} dx$ [2002 ,2004]
109. EVALUATE : $\int sec^3 x dx$ [2003]
110. EVALUATE : $\int (\sin^{-1} x)^2 dx$ [2002 ,2005]
111. EVALUATE : $\int e^{ax} \sin bx\, dx$ [2002]
112. EVALUATE : $\int e^x(\tan x - \log\cos x)\, dx$ [2000]
113. EVALUATE : $\int \frac{(x-3)e^x}{(x-1)^3} dx$ [2009]
114. EVALUATE : $\int \frac{(x^2+1)e^x}{(x+1)^2} dx$ [2006]
115. EVALUATE : $\int \log(2+x^2)\, dx$ [2001]
116. EVALUATE : $\int x\tan^{-1} x\, dx$ [2003]
117. EVALUATE : $\int x \log 2x\, dx$ [2007]
118. EVALUATE : $\int (\frac{1}{\log x} - \frac{1}{(\log x)^2})dx$ [2005]

2. DEFINITE INTEGRALS

1. $Evaluate : \int_4^5 e^x\, dx$
2. $Evaluate : \int_0^1 \frac{dx}{1+x^2}$
3. $Evaluate : \int_1^2 e^x (\frac{1}{x} - \frac{1}{x^2})dx$ [2002]

4. $Evaluate : \int_0^{\infty} \frac{1}{1+x^2} dx$ [2001]
5. $Evaluate : \int_0^{\pi/4} \sin 2x \sin 3x \, dx$ [2006]
6. $Evaluate : \int_1^{2} \frac{1}{x(1+\log x)^2} dx$
7. $Evaluate : \int_0^{\frac{\pi}{4}} \frac{\sin 2x}{sin^4 x+cos^4 x} dx$[2003]
8. $Prove\ that : \int_0^{1} \sin^{-1}(\frac{2x}{1+x^2}) \, dx = \frac{\pi}{2} - \log 2$. [2002]
9. $Prove\ that : \int_0^{\pi/4}(\sqrt{\tan x} + \sqrt{\cot x}) \, dx = \sqrt{2}\frac{\pi}{2}$. [2002 03 ,12]
10. $Prove\ that : \int_{\pi/4}^{\pi/2} \cos 2x \log \sin x \, dx = \frac{1}{4}(1 - \frac{\pi}{2} + \log 2)$. [2003]
11. $Prove\ that : \int_{-a}^{a} \sqrt{\frac{a-x}{a+x}} dx = a\pi$. [2008]
12. $Prove\ that : \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}} dx = \frac{\pi}{12}$. [2007]
13. $Evaluate : \int_{-5}^{5} |x + 2| \, dx.$
14. $Evaluate : \int_{-1}^{2} |x^3 - x| \, dx.$
15. $Evaluate : \int_1^{4} (|x - 1| + |x - 2| + |x - 3|) \, dx.$
16. $Evaluate : \int_{-\pi/2}^{\pi/2} (\cos|x| + \sin|x|) \, dx$.
17. $Evaluate : \int_{-\pi/4}^{3\pi/4} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}} dx$
18. $Evaluate : \int_0^{1} \log\left(\frac{1}{x} - 1\right) dx$
19. $Evaluate : \int_0^{\pi/2} \frac{cos^5 x}{sin^5 x+cos^5 x} dx$ [2000]
20. $Evaluate : \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}} dx$ [2009]
21. $Prove\ that : \int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx = \frac{\pi^2}{4}$ [2007,08]
22. $Evaluate : \int_{0.2}^{3.5} [x] \, dx$
23. $Evaluate : \int_0^{\pi} \frac{x}{a^2 cos^2 x+b^2 sin^2 x} dx$ [2003]
24. $Evaluate : \int_0^{\pi} \frac{x}{1+\sin x} dx$ [2001]
25. $Prove\ that : \int_0^{\pi} \frac{x \sin x}{1+cos^2 x} dx = \frac{\pi^2}{4}$ [2001,02,03,08]
26. $Prove\ that : \int_0^{\pi/2} \frac{1}{1+\tan x} dx = \frac{\pi}{4}$ [2003]
27. $Prove\ that : \int_0^{\pi} \frac{x \tan x}{\sec x+\tan x} dx = \frac{\pi^2}{4}$ [2008]
28. $Prove\ that : \int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2} \log 2 = \int_0^{\pi/2} \log \cos x \, dx$
29. $Prove\ that : \int_0^{\pi} \log(1 + \cos x) \, dx = -\pi \log 2$
30. $Prove\ that : \int_0^{\pi/4} \log(1 + \tan\theta) \, d\theta = \frac{\pi}{8} \log 2$ [2002,03,04,06]
31. $Prove\ that : \int_0^{1} \cot^{-1}(1 - x + x^2) \, dx = \frac{\pi}{2} - \log 2$. [2008]
32. $Prove\ that : \int_0^{2} x\sqrt{2 - x} \, dx = \frac{16}{15}\sqrt{2}$ [2007]
33. $Prove\ that : \int_{-1}^{1} sin^{107} x cos^{578} x \, dx = 0$
34. $Evaluate : \int_0^{\pi/2} \frac{sin^3 x}{sin^3 x+cos^3 x} dx$ [2000c]

35. $Evaluate : \int_0^{\pi/2} \frac{sin^7x}{sin^7x+cos^7x} dx$ [2000]

36. $Evaluate : \int_0^{\pi/2} \frac{sin^nx}{sin^nx+cos^nx} dx$

37. $Evaluate : \int_0^{\pi/2} (2 \log \sin x - \log \sin 2x)\, dx$ [2009]

38. $Evaluate : \int_{-\pi/2}^{\pi/2} sin^5x\, dx$ [2010]

39. $Evaluate : \int_0^1 \frac{1}{1+x^2} dx$ [2008]

40. $Evaluate : \int_{-\pi/4}^{\pi/4} sin^3x\, dx$ [2010]

41. $Evaluate : \int_0^1 \frac{2x}{1+x^2} dx$[2011]

42. $Evaluate : \int_0^{1/\sqrt{2}} \frac{1}{\sqrt{(1-x^2)}} dx$[2009]

43. $Evaluate : \int_0^5 |x-3|\, dx$ [2001]

44. $Find\ k\ , if \int_0^1 (3x^2 + 2x + k)\, dx = 0$ [2009]

Chapter -8
APPLICATION OF INTEGRALS

LEARNING OBJECTIVES/OUTCOMES

1. Standard equations of straight lines
2. Equation of circles with Centre at the origin, and center at (h,k)
3. Equation of parabolas
4. Equation of ellipse
5. First fundamental theorem of integral calculus
6. Second fundamental theorem of integral calculus

1.Area under the curve y=f(x)and the x-axis and the ordinates at x=a and at x=b is $\int_a^b ydx.$

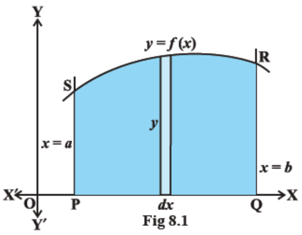
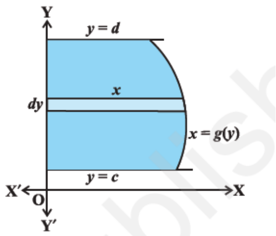
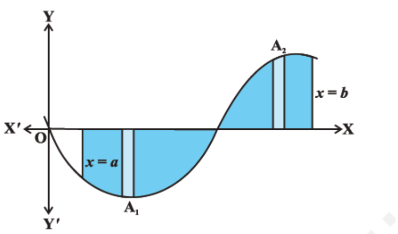


Fig 8.1

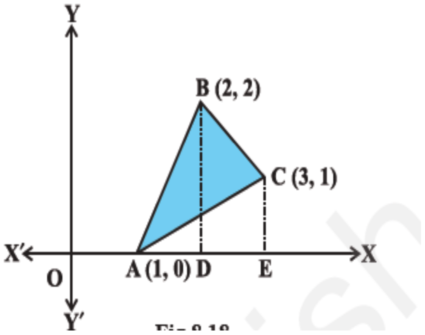
2.Area under the curve x=f(y)and the x-axis and the ordinates at y=c and at y= d is $\int_c^d xdy$



3.The area bounded by the curve y=f(x) and x-axis and the ordinates x=a and x=b is given by $A_1 + A_2 = A$



6.Area of a triangle when the coordinates of the vertices or equations of sides are given.



STEPS FOR FINDING THE AREA USING INTEGRATION

STEPS

- DRAW THE DIAGRAM
- MAKE A SHADED REGION
- FIND INTERSECTION POINTS
- IDENTIFY THE LIMITS
- WRITE THE INTEGRAL(S) FOR THE REGION
- EVALUATE THE INTEGRAL
- THE VALUE SHOULD BE POSITIVE

THREE LEVELS OF GRADED QUESTIONS

LEVEL I

1. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$ and $x = 4$ and the x - axis.

2. Find the area of the region bounded by $y = x^2$ and the lines $y = 2$ and $y = 4$ andthe y- axis.

3. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

4. Find the area of the region bounded by the parabola $y=x^2$ and $y=x$

5. Find the area of the region bounded by the curve y = x between x = -1 and x=1.

6. Find the area of the region bounded by the curve x^2=4y and the line x=4y-2

7.Find the area of the region bounded by the curve y^2= 4x and the line x= 3

8 .Find the area of the region lying in the first quadrant and bounded by y=4x^2, x=0,y=1 and y=4

9. Find the area of the region enclosed by the parabola x^2 = y, and the line y = x+2 and the x-axis.

10. Find the area lying in the first quadrant and bounded by the circle x^2+ y^2=4 and the lines x=0 and x=2.

11. Find the area of the region bounded by the curve $y = \frac{1}{x}, x -$ axis and between $x = 1, x = 4$

12 Find the area of the region bounded by the curve $y = x + 1$ and the lines $x = 2\ and\ x = 3$

13 Find the area of the region bounded by the curve $y = cosx,$ between $x = 0\ and\ x = \pi$

14 Find the area bounded by the curve $x^2 + y^2 = 4$ in first quadrant

LEVEL 2

1. Find the area of the region bounded by the parabola y= $x^2$and y = $|x|$

2. Using Integration find the area of the region bounded by the triangle whose vertices are (1,0), (2,2) and (3,1)

3. Using integration find the area of the triangular region whose sides have the equations y = 2x + 1, y = 3x + 1 and x = 4

4. Find the area bounded by the curve y = sin x between x = 0 and x =2 π

5. Find the area of the smaller region enclosed by the circle $x^2 + y^2 = 4$ and the line

$$x + y = 2$$

6. Find the area of the parabola y^2=4ax bounded by the latus rectum.

7. Find the area of the region bounded by the line y=3x+2, the x-axis and the ordinates x=-1 and x=1

8. Find the area of the region included between $4y=3x^2$, and the line 3x-2y+12=0

.

9. Find area of the region { (x, y) :$y \leq x^2 +1$, $y \leq x+1$, $0 \leq x \leq 2$}

10. Find the area enclosed by the curve $\frac{x^2}{9}+\frac{y^2}{16}=1$

11 Find the smaller of the two areas enclosed between the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $bx+ay=ab$

12. Find the smaller of two areas bounded by the curve $y=|x|$ and $x^2+y^2=8$

LEVEL 3

1. Sketch the region bounded by the curves y= $\sqrt{5-x^2}$and y= $|x-1|$ and find its area.

2. Make a rough sketch of the region given below and find its area using integration { (x, y); $0 \leq y \leq x^2$, $0 \leq y \leq 2x+3$, $0 \leq x \leq 3$ }

3. Sketch the graph of $y=|x+1|$. Evaluate $\int_{-3}^{1}|x+1|dx$

4. Sketch the graph of

$f(x)=\begin{cases}|x-2|+2, x \leq 2\\ x^2-2, \ x>2\end{cases}$ Evaluate $\int$ f(x)dx

5. Find the area bounded by the curve $y=x^3$, the x-axis and the ordinates x=-2 and x=1

6. Using integration , find the area of the region bounded by the triangle whose vertices are $(-2,2)(0,5) and \ (3,2)$

7. Find the area bounded by curves $y=|x|-1$ and $y=1-|x|$

8. Find the area bounded by the curves $y=|x-1|$and $y=3-|x|$

9. Find the area bounded by curve $y=x|x|$ and x -axis and the ordinates $x=-1$ and $x=1$

Chapter-9

DIFFERENTIAL EQUATIONS

Order of a differential equations- The order of the highest order derivative in the differential equation is the order of a differential equation.

Degree of a differential equations – The power/degree of the highest order derivative, when the differential coefficients are made free from the radicals and the fractions, is the degree/power of a differential equation.

Examples

1. Write the degree of the differential equation:
   $x^3(\frac{d^2y}{dx^2})^2 + x(\frac{dy}{dx})^4 = 0$ Ans. 2
2. Write the sum of the order and degree of the differential equation:
   $1 + (\frac{dy}{dx})^4 = 7(\frac{d^2y}{dx^2})^3$ Ans. 2+3 = 5

General and Particular Solutions of a Differential Equation – The solution which contains arbitrary constants is called the general solution of the differential equation.

The solution free from arbitrary constants is called a particular solution of the differential equation.

The number of arbitrary constants in general solution of a differential equation of n^{th} order are n.

Solution of Differential Equations –

(i) **Solution of differential equations by variable separable method** –
A variable separable form of the differential equation is the one which can be expressed in the form of f(x)dx = g(y)dy. The solution is given by
$\int f(x)dx = \int g(y)dy + k$, where k is the constant of integration.

(ii) **Solution of Homogeneous differential equations** –
Identifying a Homogeneous differential equation:
$\frac{dy}{dx} = f(x,y)$, If $f(kx,ky) = k^n f(x,y)$, then the given differential equation is homogeneous of degree n
Putting y = vx therefore $\frac{dy}{dx} = v + x\frac{dv}{dx}$
We seperate the variables to get the required solution.

(iii) **Solution of Linear differential equation**:
Standard form of Linear differential equation is $\frac{dy}{dx} + P(x)y = Q(x)$, where P(x) and Q(x) are functions of x only.
Integrating factor (IF)= $e\int P(x)dx$
The solution is given by
$y.\ IF = \int[Q(x)dx.IF]dx + c$

S.No.	QUESTIONS ON ORDER AND DEGREE OF A DIFFERENTIAL EQUATION Find the order and degree (if defined) of the following differential equations
Q.1	$\left(\frac{d^2y}{dx^2}\right)^2 + \cos(\frac{dy}{dx}) = 0$
Q.2.	$\left(\frac{d^2s}{dt^2}\right)^2 + \left(\frac{ds}{dt}\right)^3 + 4 = 0$
Q.3.	$\left(\frac{dy}{dx}\right)^4 + 3y\left(\frac{d^2y}{dx^2}\right) = 0$
Q.4	$\left(\frac{d^2y}{dx^2}\right) + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \log x$
Q.5	$\frac{dy}{dx} + \sin(\frac{dy}{dx}) = 0$
Q.6	$\left(\frac{d^3y}{dx^3}\right) + 2\left(\frac{d^2y}{dx^2}\right) + \frac{dy}{dx} = 0$
Q.7	$2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$
Q.8	$\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - \sin^2 x = 0$

S.No.	QUESTIONS ON SOLVING LINEAR DIFFERENTIAL EQUATION Solve the following differential equations
Q.1	$\frac{dy}{dx} + \frac{y}{x} = x^2$
Q.2.	$x\frac{dy}{dx} + 2y = x^2 \log x$
Q.3.	$x\frac{dy}{dx} + y - x + xy\, cot x = 0$
Q.4	$x\frac{dy}{dx} + y - x + xy \cot x = 0$
Q.5	$(x+y)\frac{dy}{dx} = 1$
Q.6	Find a particular solution : $\frac{dy}{dx} + 2y\tan x = sinx\ ; y = 0\ when\ x = \frac{\pi}{3}$
Q.7	Find a particular solution : $\frac{dy}{dx} - 3ycot\ x = \sin 2x\ ; y = 2\ when\ x = \frac{\pi}{2}$
Q.8	Find a particular solution : $\frac{dy}{dx} + cotx = 4x\ cosec\ x\ ; y = 0\ whenx = \frac{\pi}{2}$

S.No.	QUESTIONS ON SOLVING HOMOGENEOUS DIFFERENTIAL EQUATION Solve the following differential equations
Q.1	$x\frac{dy}{dx} = x + y$
Q.2.	$(x^2 - y^2)dx + 2xydy = 0$
Q.3.	$2xydx + (x^2 + 2y^2)dy = 0$
Q.4	$\frac{dy}{dx} + \frac{x^2 - y^2}{3xy} = 0$
Q.5	$x\frac{dy}{dx} - y + x\, sin\frac{y}{x} = 0$
Q.6	Find a particular solution : $2xy + y^2 - 2x^2\frac{dy}{dx} = 0$; y = 2 when x = 1
Q.7	Find a particular solution : $\left[x\, sin^2\left(\frac{y}{x}\right) - y\right]dx + x\,dy = 0\ ; y = \frac{\pi}{4}\ when\ x = 1$

Chapter-10
VECTOR ALGEBRA

KEY POINTS:

Position vector of a point:

If P(x,y,z) be any point in space then its position vector with respect to point O is $\overrightarrow{OP}$ and is usually denoted by $\vec{r}$ and is given by $\overrightarrow{OP} or \vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Magnitude of a vector:

Length of a vector is called its magnitude. Magnitude of a vector $\vec{r}$ is denoted by $|\vec{r}|$ or r and is given by $|\vec{r}|$ or r=$\sqrt{x^2 + y^2 + z^2}$

Negative of a Vectors :

A vector whose magnitude is the same as that of a given vector say $\overrightarrow{AB}$, but direction is opposite to that of it, is called negative of the given vector. Thus vector $\overrightarrow{BA}$ is negative of vector$\overrightarrow{AB}$. It is written as

$$\overrightarrow{BA} = -\overrightarrow{AB}$$

Unit vector along a given vector:

Vector with unit(one) magnitude is called unit vector. Unit vector along (in the direction of) vector $\vec{a}$is denoted by $\hat{a}$ and is given by $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$.

Vector of given magnitude in the direction of a vector:

Vector of magnitude λ in the direction of vector $\vec{a} = \lambda\hat{a}$

Direction cosine and direction ratios of a vector:

If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ makes angle α, β, γ with positive direction of X,Y,Z axes, then $\cos\alpha, \cos\beta, \cos\gamma$ are called direction cosines of vector $\vec{r}$and are denoted by l,m,n

Hence $l = \cos\alpha, m = \cos\beta, n = \cos\gamma$

Also $l = \frac{x}{r}, m = \frac{y}{r}, n = \frac{z}{r}$ where r is magnitude of $\vec{r}$

Ratios of direction cosines $l, m, n$is called direction ratios and are usually denoted by a,b,c.

In case of vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ we may take x,y,z as direction ratios of vector $\vec{r}$

Projection of $\vec{a} on \vec{b}$:

Projection of $\vec{a} on \vec{b} = \vec{a}.\hat{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

Collinear vectors:

Two vectors are called collinear if they are parallel to each other, means either they are in same direction or they are in opposite direction.

$$\vec{a} and \vec{b} will be collinear if \vec{a} = \lambda\vec{b}$$

Also if $\vec{a} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k} and \vec{b} = a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k}$then they will be collinear if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Vector joining two points:

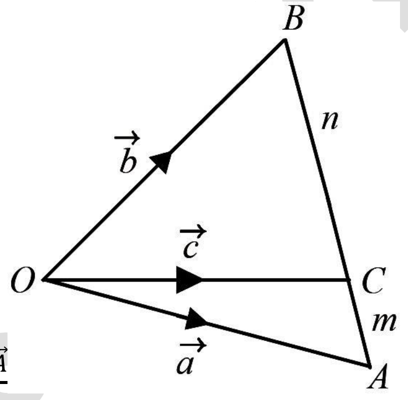
If A(x_1, y_1, z_1) and B(x_2, y_2, z_2) be any two points then vector $\overrightarrow{AB}$ is given by

$$\overrightarrow{AB} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$$

Section formula:

1. **Internal Division:**

Position vector of a point C dividing a vector $\overrightarrow{AB}$internally in the ratio of m : n is given by



$\overrightarrow{OC} = \frac{m\overrightarrow{OB} + n\overrightarrow{OA}}{m+n}$

2. **External Division** :

Position vector of a point C dividing a vector $\overrightarrow{AB}$externally in the ratio of m : n is given by $\overrightarrow{OC} = \frac{m\overrightarrow{OB} - n\overrightarrow{OA}}{m-n}$

3. Mid Point formula:

If C be the mid point of $\overrightarrow{AB}$ then $\overrightarrow{OC} = \frac{\overrightarrow{OA} + \overrightarrow{OB}}{2}$

Product of two vectors:

Scalar (or dot) product of two vectors

The scalar product of two nonzero vectors $\vec{a} and \vec{b} is$ denoted by $\vec{a}\,.\,\vec{b}$, is defined as as $\vec{a}\,.\,\vec{b}$ $=|\vec{a}||\vec{b}|$ cos θwhere, θ is the angle between $\vec{a} and \vec{b}$, 0 ≤ θ ≤π

If $\vec{a} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k} and \vec{b} = a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k}$ then

$\vec{a}\,.\,\vec{b}$ $=a_1a_2 + b_1b_2 + c_1c_2$

Important results:

1. $\vec{a}\,.\,\vec{b} is scalar (real number)$
2. $\vec{a}\,.\,\vec{b}$=0 *if and only if* $\vec{a} and \vec{b} are perpendicular.$
3. $\vec{a}\,.\,\vec{a} = |\vec{a}|^2$
4. $\cos\theta = \frac{\vec{a}\,.\vec{b}}{|\vec{a}||\vec{b}|}$
5. The scalar product is commutative. i.e. $\vec{a}\,.\,\vec{b} = \vec{b}.\vec{a}$
6. $\hat{\imath}.\hat{\imath} = 1, \hat{\jmath}.\hat{\jmath} = 1, \hat{k}.\hat{k} = 1$
7. $\hat{\imath}.\hat{\jmath} = 0, \hat{\jmath}.\hat{k} = 0, \hat{k}.\hat{\imath} = 0$

Vector (or cross) product of two vectors

The vector product of two nonzero vectors $\vec{a} and \vec{b} is$ denoted by $\vec{a} \times \vec{b}$, is defined as as $\vec{a} \times \vec{b}$ $=|\vec{a}||\vec{b}|$ sin θn̂where, θ is the angle between $\vec{a} and \vec{b}$, 0 ≤ θ ≤π and n̂ is unit vector perpendicular to both $\vec{a} and \vec{b}$

If $\vec{a} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k} and \vec{b} = a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k}$then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

Important results:

1. *$\vec{a} \times \vec{b}$ is a vector and it is perpendicular to both $\vec{a} and \vec{b}$*
2. *Vector perpendicular to both both $\vec{a} and \vec{b} = \lambda\vec{a} \times \vec{b}$*
3. *Unit vector perpendicular to both both $\vec{a} and \vec{b}$=$\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$*
4. $\vec{a} \times \vec{b}$ =0 *if and only if* $\vec{a} and \vec{b} are parallel (collinear)$.
5. $\vec{a} \times \vec{a}$=0
6. $\vec{a} \times \vec{b} = \overrightarrow{-b} \times \vec{a}$
7. *sin* θ=$\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

8. $\hat{\imath} \times \hat{\imath} = 0, \hat{\jmath} \times \hat{\jmath} = 0, \hat{k} \times \hat{k} = 0$
9. $\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$
10. $\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}, \hat{\imath} \times \hat{k} = -\hat{\jmath}$
11. If $\vec{a} and \vec{b}$ represent two adjacent sides of a triangle then area of that triangle $=\frac{1}{2}\left|\vec{a} \times \vec{b}\right|$
12. If $\vec{a} and \vec{b}$ represents two adjacent sides of a parallelogram then area of the parallelogram $=\left|\vec{a} \times \vec{b}\right|$
13. If $\overrightarrow{d_1}$ *and* $\overrightarrow{d_2}$represent two diagonals a parallelogram then area of the parallelogram $=\frac{1}{2}\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right|$

IMPORTANT PROBLEMS
(1 Mark Each)

1. Find a unit vector in the direction of the vector $\vec{b} = 6\hat{i} - 2\hat{j} + 3\hat{k}$

   Solution. Unit vector in the direction of $\vec{b}$ will be $\frac{\vec{b}}{|\vec{b}|}$

$$= \frac{6\hat{i} - 2\hat{j} + 3\hat{k}}{\sqrt{6^2 + (-2)^2 + (3)^2}} = \frac{1}{7}(6\hat{i} - 2\hat{j} + 3\hat{k})$$

2. Find the projection of $\vec{a}$ on $\vec{b}$, if $\vec{a}.\vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

   Solution. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$$= \frac{8}{\sqrt{2^2 + 6^2 + 3^2}} = \frac{8}{7}$$

3. Write the value of p for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors.

   Solution. Two vectors are parallel if their direction ratios are proportional i.e. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\therefore \quad \frac{3}{1} = \frac{2}{p} = \frac{9}{3} \text{ or } 3p = 2 \text{ or } p = 2/3$$

4. Find the value of p if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = 0$

   Solution. Cross product of the given two vectors =0

$$\Rightarrow \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & 3 & p \end{vmatrix} = 0$$

$$\Rightarrow \quad i(6p - 81) - \hat{j}(2p - 27) + \hat{k}(0) = \vec{0}$$

$\Rightarrow \quad 6p-81=0 \quad$ or $\quad p=\dfrac{81}{6}$ or $p=\dfrac{27}{2}$

5. If $\vec{p}$ is a unit vector and $(\vec{x}-\vec{p}).(\vec{x}+\vec{p})=80$ find $|\vec{x}|$

   Solution. $\quad (\vec{x}-\vec{p}).(\vec{x}+\vec{p})=80$

   $\Rightarrow \quad \vec{x}^2-\vec{p}^2=80$

   or $\quad |\vec{x}|^2-|\vec{p}|^2=80 \quad$ since $|\vec{p}|^2=1$

   $\therefore \quad |\vec{x}|^2-1=80 \quad$ or $\quad |\vec{x}|^2=81 \quad \therefore \quad |\vec{x}|=9$

6. Find the angle between the vectors $\vec{a}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}-\hat{k}$

   Solution. $\therefore \quad \vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

   $\therefore \quad \cos\theta=\dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

   $$=\frac{(\hat{i}-\hat{j}+\hat{k}).(\hat{i}+\hat{j}-\hat{k})}{\sqrt{1^2+1^2+1^2}\sqrt{1^2+1^2+1^2}}=\frac{1-1-1}{\sqrt{3}.\sqrt{3}}$$

   $$=\frac{-1}{3}$$

   $$\theta=\cos^{-1}\left(-\tfrac{1}{3}\right)$$

7. For what value of λ are the vector $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$ perpendicular to each other.

   Solution. If $\vec{a}$ are $\vec{b}$ are perpendicular then $\vec{a}.\vec{b}=0$

   $\therefore \quad 2(1)+\lambda(-2)+1(3)=0$

   $-2\lambda=-5 \quad$ or $\quad \lambda=\tfrac{5}{2}$

8. Find a vector in the direction of vector $\vec{a}=\hat{i}-2\hat{j}$ whose magnitude is 7.

   Solution. Unit vector in the direction of $\vec{a}=\dfrac{\vec{a}}{|\vec{a}|}$

   $$=\frac{\hat{i}-2\hat{j}}{\sqrt{1^2+(-2)^2}}=\frac{1(\hat{i}-2\hat{j})}{\sqrt{5}}$$

   Now a vector whose magnitude is 7 will be $\dfrac{7}{\sqrt{5}}(\hat{i}-2\hat{j})$

(2 Marks Each)

1. If $\vec{a}=\hat{i}+\hat{j}$; $\vec{b}=\hat{j}+\hat{k}$; $\vec{c}=\hat{k}+\hat{i}$, find a unit vector in the direction of $\vec{a}+\vec{b}+\vec{c}$

Solution. $\vec{a}+\vec{b}+\vec{c}=2\hat{i}+2\hat{j}+2\hat{k}$

$\therefore$ Unit vector in the direction of $\vec{a}+\vec{b}+\vec{c}=\dfrac{\vec{a}+\vec{b}+\vec{c}}{|\vec{a}+\vec{b}+\vec{c}|}$

$$= \frac{2(\hat{i}+\hat{j}+\hat{k})}{\sqrt{2^2+2^2+2^2}}$$

$$= \frac{2(\hat{i}+\hat{j}+\hat{k})}{2\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$$

2. What is the angle between vectors $\vec{a}$ and $\vec{b}$ with magnitude $\sqrt{3}$ and 2 respectively? Given $\vec{a}.\vec{b} = 3$.

Solution. $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and $\vec{a}.\vec{b} - 3$ Let θ be the angle between $\vec{a}$ and $\vec{b}$

Now $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{3}{\sqrt{3}\times 2} = \frac{3\times\sqrt{3}}{\sqrt{3}\times 2\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\therefore \qquad \theta = \pi/6$

3. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|=3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$ and $\vec{a}\times\vec{b}$ is a unit vector.

Then what is the angle between $\vec{a}$ and $\vec{b}$.

Solution. $\therefore \vec{a}\times\vec{b}$ is a unit vector $\therefore |\vec{a}\times\vec{b}|=1$

$\Rightarrow |\vec{a}|.|\vec{b}|\sin\theta = 1$, where '$\theta$' is the angle between $\vec{a}$ and $\vec{b}$.

$\Rightarrow 3\times\frac{\sqrt{2}}{3}\sin\theta = 1 \quad \Rightarrow \sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \pi/4$

4. Write the value of $\hat{i}.(\hat{j}\times\hat{k}) + \hat{j}.(\hat{k}\times\hat{i}) + \hat{k}.(\hat{j}\times\hat{i})$

Solution. $\hat{j}\times\hat{k} = \hat{i}, \ \hat{k}\times\hat{i} = \hat{j}$ and $\hat{i}\times\hat{j} = \hat{k}$

Therefore $\hat{i}.(\hat{j}\times\hat{k}) + \hat{j}.(\hat{k}\times\hat{i}) + \hat{k}.(\hat{j}\times\hat{i}) = \hat{\imath}.\hat{\imath} + \hat{\jmath}.\hat{\jmath} - \hat{k}.\hat{k}$

$= 1 + 1 - 1 = 1$

(3 Marks Each)

1. If $\vec{a} = 2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}$ are such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$. Find the value of λ

Solution. $\vec{a}+\lambda\vec{b} = (2\hat{i}+2\hat{j}+3\hat{k}) + \lambda(-\hat{i}+2\hat{j}+\hat{k})$

$= (2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}$

Now $(\vec{a}+\lambda\vec{b})$ is perpendicular to $\vec{c}$.

$\therefore \qquad (\vec{a}+\lambda\vec{b}).\vec{c} = 0$

$\Rightarrow$ $[(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}].(3\hat{i}+\hat{j})=0$

or $(2-\lambda).3+(2+2\lambda).1+(3+\lambda).0=0$

or $8-\lambda=0$ *or* $\lambda=8$

2. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$ where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$

Solution. Two non-zero vectors are parallel if and only if their vector product is a zero vector.

Here $(\vec{a}-\vec{d})\times(\vec{b}-\vec{c})=(\vec{a}\times\vec{b}-\vec{a}\times\vec{c}-\vec{d}\times\vec{b}+\vec{d}\times\vec{c})$

$$=\vec{c}\times\vec{d}-\vec{b}\times\vec{d}+\vec{b}\times\vec{d}-\vec{c}\times\vec{d}$$

$$=\vec{0}$$ (Using given results)

Hence $(\vec{a}-\vec{d})$ is parallel to $(\vec{b}-\vec{c})$.

3. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

Solution. Unit vector along the sum of vectors = $\dfrac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}$

$$=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+(6)^2+(-2)^2}}=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$$

It is given that the dot product of this vector with $(\hat{i}+\hat{j}+\hat{k})$ is 1.

$$\therefore\quad \frac{2+\lambda}{\sqrt{\lambda^2+4\lambda+44}}.1+\frac{6}{\sqrt{\lambda^2+4\lambda+44}}.1-\frac{2}{\sqrt{\lambda^2+4\lambda+44}}=1$$

or $2\times\lambda+6-2=\sqrt{\lambda^2+4\lambda+44}$

$(\lambda+6)^2=\lambda^2+4\lambda+44$

or $8\lambda=8$

$\lambda=1$

4. If $\vec{a}.\vec{b}$ and $\vec{c}$ are vectors such that $\vec{a}.\vec{b}=\vec{a}.\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$, $\vec{a}\neq 0$, then prove that $\vec{b}=\vec{c}$

Soln. If $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ then $\vec{a}.(\vec{b}-\vec{c}) = 0$

$$\Rightarrow \quad \vec{a} = \vec{0},\ \vec{b}-\vec{c} = 0 \text{ or } \vec{a} \perp (\vec{b}-\vec{c})$$

As, $\quad \vec{a} \neq 0 \Rightarrow \vec{b}-\vec{c}$ or $\vec{a} \perp (\vec{b}-\vec{c})$ $\qquad$ (i)

Also, it is given that $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$

$$\Rightarrow \quad \vec{a}\times(\vec{b}-\vec{c}) = \vec{0}$$

$$\Rightarrow \quad \vec{a} \neq \vec{0},\ \vec{b}-\vec{c} \text{ or } \vec{a} \parallel (\vec{b}-\vec{c})$$

as $\quad \vec{a} \neq \vec{0} \Rightarrow \vec{b} = \vec{c}$ or $\vec{a} \parallel (\vec{b}-\vec{c})$ $\qquad$ (ii)

from (i) and (ii) , we get

$\vec{b} = \vec{c}$, as $\vec{a}$ cannot be parallel and perpendicular to $(\vec{b}-\vec{c})$ simultaneously.

5. Find a vector of magnitude 5 units, perpendicular to each of the vectors $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ where $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+3\hat{k}$

Solution. Let $\vec{c} = \vec{a}+\vec{b} = (\hat{i}+\hat{j}+\hat{k})+(\hat{i}+2\hat{j}+3\hat{k})$

$$= 2\hat{i}+3\hat{j}+4\hat{k}$$

Let $\quad \vec{d} = \vec{a}-\vec{b} = (\hat{i}+\hat{j}+\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})$

$$= -\hat{j}-2\hat{k}$$

Now a vector $\perp$ to both $\vec{c}$ and $\vec{d}$ will be

$$\vec{c}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = \hat{i}(-6+4)-\hat{j}(-4-0)+\hat{k}(-2+0)$$

$$= 2\hat{i}+4\hat{j}-2\hat{k}$$

And a vector of magnitude 5 units will be $= \dfrac{5(\vec{c}\times\vec{d})}{|\vec{c}\times\vec{d}|}$

$$= \frac{5(-2\hat{i}+4\hat{j}-2\hat{k})}{\sqrt{(-2)^2+(4)^2+(-2)^2}} = \frac{-2\times5(\hat{i}-2\hat{j}+\hat{k})}{2\sqrt{6}}$$

$$=-\frac{5}{6}(\hat{i}-2\hat{j}+\hat{k})$$

6. If vectors $\vec{a},\vec{b}$ and $\vec{c}$ are such that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=3, |\vec{b}|=5$ and $|\vec{c}|=7$, find the angle between $\vec{a}$ and $\vec{b}$.

Solution. $\because \quad \vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\because \quad \vec{a}+\vec{b}=-\vec{c}$

$= \quad (\vec{a}+\vec{b}).(\vec{a}+\vec{b})=(-\vec{c}).(-\vec{c})$

or $\quad |\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b}=|\vec{c}|^2$

or $\quad (3)^2+(5)^2+2(3)(5)\cos\theta=49$

$$30\cos\theta=15$$

$$\cos\theta=1/2 \text{ or } \theta=\pi/3$$

7. If $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ represents two adjacent sides of a parallelogram. Find the area of parallelogram.

Solution. If $\vec{a} and \vec{b}$ represents two adjacent sides of a parallelogram then area of the parallelogram $=|\vec{a}\times\vec{b}|$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 2 & 3\\ -1 & 2 & 1\end{vmatrix}=\hat{\imath}(2-6)-\hat{\jmath}(2+3)+\hat{k}(4+2)=-4\hat{\imath}-5\hat{\jmath}+6\hat{k}$$

So $|\vec{a}\times\vec{b}|=\sqrt{(-4)^2+(-5)^2+6^2}=\sqrt{77}$

Hence area of the parallelogram$=\sqrt{77}\ sq\ units$.

(5 Marks Each)

1.Express the vector $\vec{a}=5\hat{i}-2\hat{j}+5\hat{k}$ as the sum of two vector such that one is parallel to the vector $\vec{b}=3\hat{i}+\hat{k}$ and the other is perpendicular to $\vec{b}$.

Solution :- Let the vector parallel to $\vec{b}$ be $\lambda\vec{b}$

i.e. $\lambda(3\hat{i}+\hat{k})$, then the other vector must be $\vec{a}-\lambda\vec{b}$.

But this vector must be perpendicular to $\vec{b}$.

$$\Rightarrow (\vec{a}-\lambda\vec{b}).\vec{b}=0$$

$\Rightarrow\ [(5\hat{i} - 2\hat{j} + 5\hat{k}) - \lambda(3\hat{i} + \hat{k})].(3\hat{i} + \hat{k}) = 0$

$\Rightarrow\ (5 - 3\lambda)\hat{i} - 2\hat{j} + (5 - \lambda)\hat{k})].(3\hat{i} + \hat{k}) = 0$

$\Rightarrow\ 3(5-3\lambda) + (-2).0 + (5-\lambda).1 = 0 \Rightarrow 15 - 9\lambda + 5 - \lambda = 0$

$\Rightarrow\ 20 = 10\lambda$

$\Rightarrow\ \lambda = 2$

Hence, the vector parallel to $\vec{b}$ is $2\vec{b}$

$= 2(3\hat{i} + \hat{k}) = 6\hat{i} + 2\hat{k}.$

and the other vector = $\vec{a} - \lambda\vec{b}$

$= (5\hat{i} - 2\hat{j} + 5\hat{k}) - 2(3\hat{i} + \hat{k})$

$= -\hat{i} - 2\hat{j} + 3\hat{k}$.

Problems for self practice

(1 Mark Each)

1. Write a vector of magnitude 7 units in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$
2. Find the direction cosine $\vec{a} = i + 2j - 3k$.
3. For what value of k vector $\vec{a} = 2\hat{i} - k\hat{j} + 3\hat{k}$ and vector $\vec{b} = \hat{i} - 3\hat{j} - 4\hat{k}$ are perpendicular to each other?
4. Find angle between the vectors $\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{b} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$
5. Find the projection of the vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$.

(2 Marks Each)

1. Find a unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$.
2. Show that the points A(1, -2, -8), B(5, 0, -2) and C(11, 3, 7) are collinear .
3. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show the vectors $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$.
4. Find the value of $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.(\hat{k} \times \hat{i}) + \hat{k}.(\hat{j} \times \hat{i})$

(3 Marks Each)

1. Find a vector of magnitude 5 units, perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

2. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$. Find the value of λ

3. Find the area of the Parallelogram having diagonals $(3\hat{i} + \hat{j} - 2\hat{k})$ and $(\hat{i} - 3\hat{j} + 4\hat{k})$

4. Find the value of p if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = 0$

Answers

(1 Mark Each)

1. $\frac{7}{3}(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$
2. $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}$
3. 10/3
4. π/2
5. $\frac{10}{\sqrt{6}}$

(2 Marks Each)

1. $\frac{-\hat{\imath}+2\hat{\jmath}-2\hat{k}}{\sqrt{6}}$

4. 1

(3 Marks Each)

1. $-\frac{5}{6}(\hat{\imath} - 2\hat{\jmath} + \hat{k})$
2. 8
3. $5\sqrt{3}$ units
4. 27/2

Chapter-11

THREE DIMENSIONAL GEOMETRY

Key Points:

1) If l, m and n are the direction cosines of a line , then $l^2+m^2+n^2=1$.
2) If a, b, c are direction ratios then $l = \frac{a}{\sqrt{a^2+b^2+c^2}}$ and so for m and n.
3) The direction ratios of a vector $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ are a, b, c.
4) The angle θ between two lines is given by $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$
5) The angle θ between two lines is given by $\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$
6) For perpendicular lines a1a2 + b1b2 + c1c2 = 0
7) For parallel lines $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$
8) The vector and Cartesian equation of a line that passes through a point parallel to a given vector is $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$
9) If θ is the angle between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ then $\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$
10) The Shortest distance between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is
$$d=\left|\frac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$$
11) The vector equation of a plane which is at a distance 'd' from the origin and $\hat{n}$ is a unit vector normal to the given plane , is $\vec{r}.\hat{n} = d$
12) The equation of a plane having a, b, c as direction ratios of the normal to the plane is $ax + by + cz = d$.
13) The equation of a plane passing through a point $A(x_1, y_1, z_1)$ and perpendicular the given line with direction ratios a, b, c is given by
$$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$$
14) Cartesian equation of a plane that passes through the intersection of two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by
$a_1x + b_1y + c_1z + d_1 + \lambda(a_2x + b_2y + c_2z + d_2) = 0$, where λ is any parameter.
15) Cartesian equation of a plane passing through three non collinear points $A(x_1, y_1, z_1)$,$B(x_2, y_2, z_2)$ and C (x_3, y_3, z_3) is given by $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$

Section-A[Questions of 1 mark]

Objective Type Questions

1. Distance of the point (α, β, γ) from y-axis is

(A) β (B) $|\beta|$ (C) $|\beta| + |\gamma|$ (D) $\sqrt{\alpha^2+\gamma^2}$

2. If the directions cosines of a line are k, k, k, then

(A) $k > 0$ (B) $0 < k < 1$ (C) $k = 1$ (D) $k = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

3. The distance of the plane $\vec{r}.\left(\frac{2}{7}\hat{\imath} + \frac{3}{7}\hat{\jmath} - \frac{6}{7}\hat{k}\right) = 1$ from the origin is

(A) 1 (B) 7 (C) $\frac{1}{7}$ (D) None of these

4. The reflection of the point (α, β, γ) in the xy-plane is
   (A) $(\alpha, \beta, 0)$ (B) $(0,0,\gamma)$ (C) $(-\alpha, -\beta, \gamma)$ (D) $(\alpha, \beta, -\gamma)$

5. The area of the quadrilateral ABCD, where A(0,4,1), B(2,3,-1), C(4,5,0) and D(2,6,2), is equal to
   (A) 9 sq. units (B) 18 sq. units (C) 27 sq. units (D) 81 sq. units

6. The locus represented by $xy + yz = 0$ is
   (A) a pair of perpendicular lines (B) a pair of parallel lines
   (C) a pair of parallel planes (D) a pair of perpendicular planes

7. A line makes equal angles with co-ordinate axis. Direction cosines of this lines are
   (A) $\pm(1,1,1)$ (B) $\pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (C) $\pm\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (D) $\pm\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

8. The equation of x-axis in space are
   (A) $x = 0, y = 0$ (B) $x = 0, z = 0$ (C) $x = 0$ (D) $y = 0, z = 0$

Objective Type Questions (Fill in the blanks)

9. A plane passes through the points (2,0,0), (0,3,0) and (0,0,4). The equation of plane is ________________.

10. The direction cosines of the vector $2\hat{\imath} + 2\hat{\jmath} - \hat{k}$ are ________________.

11. The vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-4}{2}$ is ________________.

12. The vector equation of the line through the points (3,4,-7) and (1,-1,6) is ________________.

13. The cartesian equation of the plane $\vec{r}.(\hat{\imath} + \hat{\jmath} - \hat{k}) = 2$ is ______________.

14. If a line makes angles $\frac{\pi}{2}, \frac{3\pi}{4}, \frac{\pi}{4}$ with x, y, z axis, respectively, then its direction cosines are _____________.

15. If a line makes angles α, β, γ with the positive directions of the coordinate axes, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is ________________.

16. If a line makes an angle of $\frac{\pi}{4}$ with each of y and z-axis, then the angle which it makes with x-axis is ______________.

17. If θ is the acute angle between two planes $\vec{r}.\overrightarrow{n_1} = d_1$ and $\vec{r}.\overrightarrow{n_2} = d_2$, then θ is equal to _______________.

18. The angle between two diagonals of a cube is _______________.

Short Answers type Questions

1. Write the direction-cosines of the line joining the points (1,0,0) and (0,1,1) .
2. Find the direction cosines of the line passing through the following points (-2,4,-5), (1,2,3).

3.Write the direction cosines of a line equally inclined to the three coordinate axes.

4-.Write the vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$.

5-Write the equation of a line parallel to the line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$ and passing through the point(1,2,3).

6-.Express the equation of the plane $\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$ in the Cartesian form.

7-Find the direction cosines of X axis , Y axis & Z axis ?

8- Find the direction cosines of a line which makes equal angles with the coordinate axes .

9- If al line has direction ratios -18,12,-4 then what are its direction cosines?

10- The cartesian equation of a line AB is $\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$. Write the direction ratios of a line parallel to AB.

Section-B[Questions of 2 marks]

1- Find the equation of a line which passes through the point (-4,2,-3) and is parallel to the Cartesian equations of
a line AB are:-

$$\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$$

2- Find the equation of a line which passes through the point (1,2,3) and is parallel to the vector $3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

3-Find the equation of a plane passing through the origin and perpendicular to x-axis

4-Find the equation of plane with intercepts 2, 3, 4 on the x ,y, z –axis respectively.

5--.Write the distance of plane 2x– y + 2z + 1 = 0 from the origins.

6-Find the vector and Cartesian equation of the lines that passes through the origin and (5,-2,3)

7-Determine the direction cosine of normal to the plane and the distance from the origin $2x+3y-z = 5$

8- Find the vector equation of the plane which is at a distance of 7 units from the origin and which is normal to the vector $3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}$.

Section-C[Questions of 3 marks]

1- Find the value of p so that the lines, $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

2-Find the value of λ such that the line $\frac{x-2}{9} = \frac{y-1}{\lambda} = \frac{z+3}{-6}$ is perpendicular to the plane 3x – y – 2z = 7.

3--Find the shortest distance between the two lines $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and

$\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$.

4-Find the foot of perpendicular from the point (1,2,-3) and the line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$

5- Find the vector equation of the line passing through the point (1, 2, − 4) and perpendicular to the two lines:

$$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$

6-- Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ-plane .

7- Find the equation of the plane that passes through three points. (1, 1, −1), (6, 4, −5),
(−4, −2, 3)

8- Find the coordinates of the point where the line through (3, −4, −5) and (2, − 3, 1) crosses the plane $2x + y + z = 7$.

Section-D[Questions of 5 marks]

1. Find the equation of the plane passing through the point (–1, 3, 2) and perpendicular to each of the planes x + 2y + 3z = 5 and 3x + 3y + z = 0.
2. Find the vector equation of the line passing through the point (1, 2, 3) and parallel to the planes $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=5$ and $\vec{r}.(3\hat{i}+\hat{j}+\hat{k})=6$.

3--Find the image of the point(-1,-1,3) in the plane 2x+3y-4z-10=0.

4- Find the equation of the plane through the intersection of the planes 3x –y+2z-4 = 0 and x +y+z-2 = 0 and the
point (2,2,1

5- Find the shortest distance between the lines $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and equation of shortest distance.

6--Find the equation of a plane passing through the points (2, - 3, 1), (5, 2,-1) and perpendicular to the plane x – 4y +5z – 2 = 0.

Chapter-12
LINEAR PROGRAMMING

Key Points:

- Two or more linear inequations are said to constitute a **system of linear inequations.**
- The solution set of a system of linear inequations is defined as the intersection of solution sets oflinear inequations in the system.
- A linear inequation is also called a **linear constraint** as it restricts the freedom of choice of thevalues *x* and *y*.

LINEAR PROGRAMMING

In **linear programming** we deal with the optimization (maximization or minimization) of a linear function of a number of variables subject to a number of restrictions (or constraints) on variables, in the form of linear inequations in the variable of the optimization function.

A Linear Programming Problem is one that is concerned with finding the **optimal value** (maximum or minimum value) of a linear function (called **objective function**) of several variables (say *x* and *y*), subject to the conditions that the variables are **non-negative** and satisfy a set of linear inequalities (called **linear constraints).**

The term **linear** implies that all the mathematical relations used in the problem are **linear relations** while the term programming refers to the method of determining a particular **programme**or plan ofaction.

Objective function Linear function $Z = ax + by$, where *a*, *b* are constants, which has to be maximised or minimized is called a linear **objective function.** Variables *x* and *y* are called **decision variables**.

Constraints The linear inequalities or equations or restrictions on the variables of a linear programming problem are called **constraints**. The conditions $x \geq 0$, $y \geq 0$ are called non-negative restrictions.

Optimisation problem A problem which seeks to maximise or minimise a linear function (say of two variables *x* and *y*) subject to certain constraints as determined by a set of linear inequalities is called an **optimisation problem**. Linear programming problems are special type of optimisation problems.

GRAPHICAL METHOD OF SOLVING LINEAR PROGRAMMING PROBLEMS

Feasible region The common region determined by all the constraints including non-negative Constraints $x,y \geq 0$ of a linear programming problem is called the **feasible region** (or solution region) for the problem. The region other than feasible region is called an **infeasible region**.

Feasible solutions Points within and on the boundary of the feasible region represent feasible solutions of the constraints.

Any point outside the feasible region is called an **infeasible solution.**

Optimal (feasible) solution: Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an **optimal solution.**

Theorem 1 Let R be the feasible region (convex polygon) for a linear programming problem and let Z = $ax + by$ be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

A **corner point** of a feasible region is a point in the region which is the intersection of two boundary lines.

Theorem 2 Let R be the feasible region for a linear programming problem, and let Z = $ax + by$ be the objective function. If R is **bounded**, then the objective function Z has both a **maximum** and a **minimum** value on R and each of these occurs at a corner point (vertex) of R.

A feasible region of a system of linear inequalities is said to be **bounded** if it can be enclosed within lines

DIFFERENT TYPES OF LINEAR PROGRAMMING PROBLEMS

A few important linear programming problems are listed below:

1. Manufacturing problems In these problems, we determine the number of units of different products which should be produced and sold by a firm when each product requires a fixed manpower, machine hours, labour hour per unit of product, warehouse space per unit of the output etc., in order to make maximum profit.

2. Diet problems In these problems, we determine the amount of different kinds of constituents/nutrients which should be included in a diet so as to minimise the cost of the desired diet such that it contains a certain minimum amount of each constituent/nutrients.

Linear Programming Problems

Question: Solve the following linear programming problem graphically:

Minimize Z = 200 x + 500 y

subject to the constraints:

$x + 2y \geq 10$

$3x + 4y \leq 24$

$x \geq 0, y \geq 0$

Solution:

Given objective function is:

Minimize Z = 200 x + 500 y ….(i)

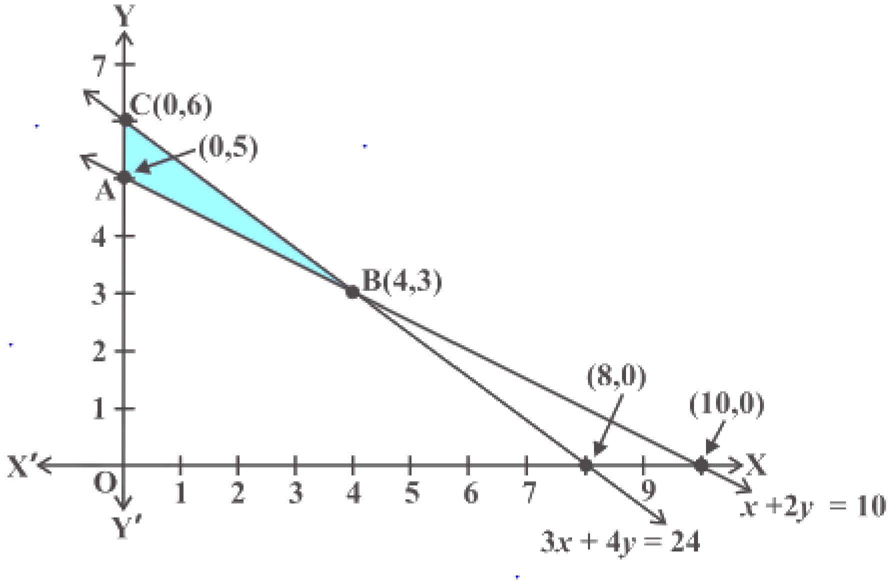
Constraints are:

$x + 2y \geq 10$ ….(ii)

3x + 4y ≤ 24 ….(iii)

x ≥ 0, y ≥ 0 ….(iv)

The graph of these inequalities is:



The shaded region in the figure (above graph) is the feasible region ABC determined by the system of constraints (ii), (iii) and (iv), which is bounded.

The coordinates of corner points of this (feasible or shaded) region say A, B and C are (0, 5), (4, 3) and (0, 6) respectively.

Now, let us evaluate the value of Z = 200x + 500y at these points.

Corner point	Corresponding value of Z
(0, 5)	200 × 0 + 500 × 5 = 0 + 2500 = 2500
(4, 3)	200 × 4 + 500 × 3 = 800 + 1500 = 2300 (minimum)
(0, 6)	200 × 0 + 500 × 6 = 0 + 3000 = 3000

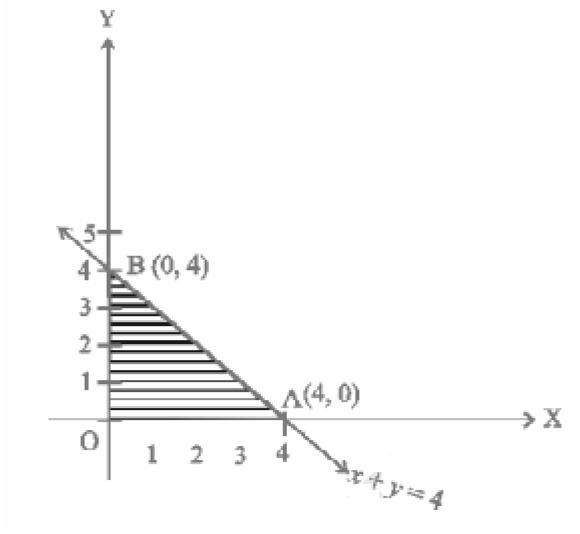
Hence, the minimum value of Z is 2300 at the point (4, 3).

Question: Solve the following LPP graphically:

Maximise Z = 2x + 3y, subject to x + y ≤ 4, x ≥ 0, y ≥ 0

Solution:

Let us draw the graph pf x + y =4 as below.



The shaded region (OAB) in the above figure is the feasible region determined by the system of constraints $x \geq 0$, $y \geq 0$ and $x + y \leq 4$.

The feasible region OAB is bounded and the maximum value will occur at a corner point of the feasible region.

Corner Points are O(0, 0), A (4, 0) and B (0, 4).

Evaluate Z at each of these corner points.

Corner Point	Value of Z
O(0, 0)	2 (0) + 3(0) = 0
A (4, 0)	2 (4) + 3(0) = 8
B (0, 4)	2 (0) + 3 (4)= 12 ← maximum

Hence, the maximum value of Z is 12 at the point (0, 4).

Question: A manufacturing company makes two types of television sets; one is black and white and the other is colour. The company has resources to make at most 300 sets a week. It takes Rs 1800 to make a black and white set and Rs 2700 to make a coloured set. The company can spend not more than Rs 648000 a week to make television sets. If it makes a profit of Rs 510 per black and white set and Rs 675 per coloured set, how many sets of each type should be produced so that the company has a maximum profit? Formulate this problem as a LPP given that the objective is to maximise the profit.

Solution:

Let x and y denote, respectively, the number of black and white sets and coloured sets made each week.

Thus $x \geq 0$, $y \geq 0$

The company can make at most 300 sets a week, therefore, $x + y \leq 300$.

Weekly cost (in Rs) of manufacturing the set is $1800x + 2700y$ and the company can spend up to Rs. 648000.

Therefore, $1800x + 2700y \leq 648000$

or

$2x + 3y \leq 720$

The total profit on x black and white sets and y coloured sets is Rs $(510x + 675y)$.

Let the objective function be $Z = 510x + 675y$.

Therefore, the mathematical formulation of the problem is as follows.

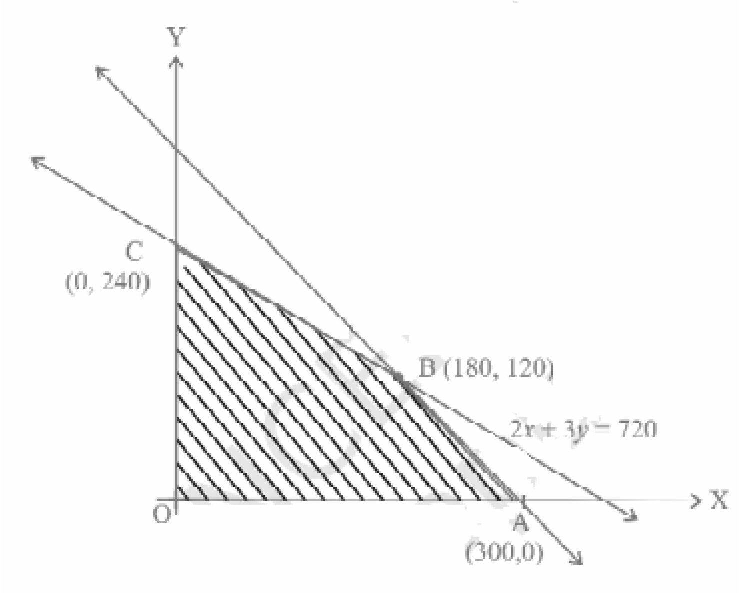
Maximise $Z = 510x + 675y$ subject to the constraints :

$x + y \leq 300$

$2x + 3y \leq 720$

$x \geq 0$, $y \geq 0$

The graph of x + y = 30 and 2x + 3y = 720 is given below.



Corner point	Value of Z
A(300, 0)	153000
B(180, 120)	172800 = Maximum
C(0, 240)	162000

Hence, the maximum profit will occur when 180 black & white sets and 120 coloured sets are produced.

Question for Practice

1) Solve the following LPP graphically. Minimize Z = 3x+5y
subject to: -2x+y≤4, x+y≥3, x-2y≤2, x,y≥0.
Ans : Minimum value is 29/3 at (8/3,1/3)
2) Solve Graphically
Maximize Z=3x+2y
Subject to: x+2y≤10, 3x+y≤15, x,y≥0.

3) Determine graphically the minimum value of the objective function. Z=-50x+20y Subject to constraints 2x-y≥-5, 3x+y≥3, 2x-3y≤12, x,y≥0

4) Solve the following LPP graphically:
Maximize Z=10x+20y
Subject to the constraints
3x + 4y ≤ 12
x ≤ 1
y ≥ 2
x ≥ 0, y ≥ 0

5) A firm makes two types of furniture: chairs and tables. The contribution to profit for each product as calculated by the accounting department is Rs.20 per chair and Rs.30 per table. Both products are to be processed on three machines M1 , M2 and M3 . The time required in hours by each product and total time available in hours per week on each machine are as follows:

Machine	Chair	Table	Available Time
M1	3	3	36
M2	5	2	50
M3	2	6	60

How should the manufacturer schedule his production in October to maximize profit.Ans: 3 chairs and 9 tables.

6) If a young man rides his motorcycle at 25 km/hr, he had to spend Rs.2 per km on petrol. If he rides at a faster speed of 40 km/hr, the petrol cost increases at Rs.5 per km. He has Rs.100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this as LPP and solve it graphically. Ans: Maximum at (50/3,40/3) and is equal to 30 km.

Additional Questions:-

1. A manufacturing company makes two types of television sets; one is black and white and other is colour. The company has resources to make at most 300 sets a week. It takes Rs 1800 to make a black and white set and Rs 2700 to make a coloured set. The company can spend not more than 648000 a weak to make television sets. If it makes a profit of Rs 510 per black and white set and Rs 675 per coloured set, how many sets of each types should be produced so that the company has maximum profit? Formulate this problem as a LPP given that the objective is to maximise the profit.

2. Minimise $Z = 3x + 5y$ subject to the constraints : $x + 2y \geq 10$ $x + y \geq 6$ $3x + y \geq 8$ $x, y \geq 0$

3. A manufacturer of electronic circuits has stock of 200 resistors, 120 transistors and 150 capacitors and is required to produce two types of circuits. A and B. Type a requires 20 resistors, 10 transistors and 10 capacitors. Type B requires 10 resistors, 20 transistors and 30 capacitors. If the profit on type A circuit is Rs. 50 and that on type B circuit is Rs 60, formulate and solve this problem as a LPP so that the manufacturer can maximise his profit.

4. A company manufactures two types of screws A and B. All the screws have to pass through a threading machine and a slotting machine. A box of types A screws requires 2 minutes on the threading machine and 3 minutes on the slotting machine. A box of type B screws requires 8 minutes of threading on the threading machine and 2 minutes on the slotting machine. In a week, each machine is available for 60 hours on selling these screws company gets a profit of Rs 100 per box on type A and Rs 170 on type B. Formulate and solve the LPP.

5. A man rides his motorcycle at the speed of 50 km/hour. He has to spend Rs 2 per km on petrol. If he rides it at a faster speed of 80 km/hour, the petrol cost increases to Rs 3 per km. He has atmostRs 120 to spend on petrol and one hour's time. He wishes to cover the maximum distance that he can travel. Express and solve this problem as a linear programming problem.

6. A manufacturer produces two model of bikes—Model X and Model Y. Model X takes a 6 man-hours to make per units, while model Y takes 10 man-hours per unit. There is total of 450 man-hours available per week. Handling and marketing costs are Rs 2000 and Rs 1000 per unit for models X and Y respectively. The total funds available for these purposes are Rs 80,000 per week. Profits per unit for models X and Y are Rs 1000 and Rs 500, respectively. How many bikes of each model should the manufacturer produce so as to yield a maximum profit? Find the maximum profit.

7. A company makes 3 models of calculators : A , B and C at factory I and factory II. The company has orders for at least 6400 calculators of model A, 4000 calculaters of model B and 4800 calculator of model C. At factory I, 50 calculators of model A, 50 of model B and 30 of model C are made every day; at factory II 40 calculaters of model A, 20 of model B and 40 of model C are made everyday. It costs Rs 1200 and Rs 15000 each day to operate factory I and II, respectively find the number of days each factory should operate to minimise the operating costs and still meet the demand.

ANSWERS

1. Maximise $Z = 510x + 675y$ subject to $x + y \leq 300$ $2x + 3y \leq 720$ $x \geq 0, y \geq 0$ Maximum profit Rs 172800 at No of B/W TV = 180, No of Coloured TV = 120

2. Z is maximum at $x = 2$, $y = 4$ maximum value of $Z = 26$

3. Maximise $Z = 50x + 60y$ subject to $2x + y \leq 20$ $x + 2y \leq 12$ $x + 3y \leq 15$ $x, y \geq 0$ Type A = 6, maximum profit = Rs 480 Type B = 3

4. Maximise $Z = 100x + 170y$ subject to $3x + 2y \leq 3600$ $3x + y \leq 600$ $x, y \geq 0$, maximum profit = Rs 138600 5. Maximise $Z = x + y$ subject to $2x + 3y \leq 120$ $8x + 5y \leq 400$ maximum distance = 54 2 7 km $x, y, > 0$

6. Model x = 25, model y = 30 Maximum profit = Rs 40000

7. Factory I = 80 days Factory II = 60 days

Chapter-13
PROBABILITY

key points

- **Sample Space**: Set of all possible outcomes in a random experiments is its Sample Space.
- **Event**: Every subset of sample space is an event.
- **Mutually exclusive events**:Two events A and B are said to be mutually exclusive if $A \cap B = \Phi$
- **Mutually exclusive and exhaustive events**:A_1 , A_2 , A_3 , ….. A_n are mutually exclusive and exhaustive events if $A_i \cap A_j = \Phi$ and $A_1 \cap A_2 \cap$ …………. $\cap A_n = S$, where S is the sample space.
- **Probability of an Event:** If A is an event of an experiment and S is the sample space, then the probability of A $_i$.e $P(A)=n(A)/n(S)$, where $0 \le P(A) \le 1$.
- **Probability of the event 'A or B':**$P(A \cup B)=P(A) + P(B) - P(A \cap B)$
- **If A and B are mutually exclusive events** then $P(A \cup B)=P(A)+P(B)$.
- **If A, B, C are three events** associated with a random experiment,
  prove that $P(A \cup B \cup C) = P(A)+P(B)+P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$.

- **If A is any event**, then P(A) + P(not A) = 1
- **Conditional probability**: Conditional probability P(A/B) = Probability that event A will occur if the event B has already occurred is
  $P(A/B) = P(A \cap B) / P(B)$ provided $P(B) \neq 0$

  $P(E'/F) = 1 - P(E/F)$

- **Multiplication theorem on Probability**:
  $P(A \cap B) = P(A)\, P(B/A) = P(B)\, P(A/B)$ provided $P(A) \neq 0$, $P(B) \neq 0$

  $P(A \cap B \cap C) = P(A).P(B/A)\, P(C/A \cap B)$

- **Independent Events**: Two events A and B are independent if
  $P(A \cap B)= P(A).P(B)$

- **Two events A and B are said to be independent**,
  if $P(A/B) = P(A)$, $P(B) \neq 0$

  and $P(B/A) = P(B)$, $P(A) \neq 0$

- **Multiplication rule when events A and B are independent**
  $P(A \cap B) = P(A).P(B)$

- 15. **Law of total probability** :
- $P(A) = P(E_1).P(A/E_1)+P(E_2).P(A/E_2) + \ldots + P(E_n)P(A/E_n)$
- 16. **Bayes' theorem**:

$P(E_1/A) = \underline{\qquad P(E_1).P(A/E_1) \qquad}$

$P(E_1).P(A/E_1)+P(E_2).P(A/E_2)+ \dots + P(E_n).P(A/E_n)$

- 17. **Random Variable**:A random variable is a real valued function whose domain is the sample space of a random experiment.
- 18. **Probability Distribution**: If a random variable X take values $x_1, x_2, x_3 \dots x_n$ with respective probabilities $p_1, p_2, p_3, \dots p_n$ then

X	x_1	x_2	x_3	….	x_n
P(X)	p_1	p_2	p_3	….	p_n

is known as the probability distribution of X.

Sum of probability of a probability distribution is always **1.**

VSA TYPE_1 MARKS QUESTIONS (SOLVED)

1. **If P(A) = 7/17, P(B) = 9/17 and P (A∩B) = 4/17 evaluate P (A/B)**

Solution : We have $P(A/B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{4/17}{9/17} = \dfrac{4}{9}$

2. **If P (A) = 4/5 and P (B) = 2/3, find P (A∩B) if A and B are independent events.**

Solution : A and B are independent events

:. P ((A∩B) = P (A). P (B)

= 4/5. 2/3 = 8/15

3. **Given** $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$ and $P(A\cap B)=\frac{1}{6}$ **. Are the event A and B independent.**

Solution. $P(A).P(B)=\frac{1}{2}\times\frac{1}{3}=P(A\cap B)$

$\therefore$ Events are in independent

4. **A die is thrown twice. Find the probability of getting a number 6 on the first throw and a number greater than 4 on the second.**

Solution. Favourable cases are { (6,5), (6,6)}

$\therefore$ Probability $= \frac{2}{36} = \frac{1}{18}$

5. An urn contains 4 white and 3 red balls. Let X be the number of red balls in a random draw of three balls .write probable value of X.

Solution: X= 0, 1, 2, 3

VSA TYPE_1 MARKS QUESTIONS(UNSOLVED)

1. If A and B are independent events, Find P(B) if P(AUB) = 0.60 and P(A) = 0.35. [**Answer : 5/13]**
2. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that the youngest is a girl. [**Answer : 1/2]**
3. Find the probability of drawing two white balls in succession from a bag containing 3 red and 5 white balls respectively, the ball first drawn is not replaced. [**Answer : 5/14**]
4. Let x represent the number of heads when a coin is tossed 6 times. What are the probable values of x**.** **[Answer : X= 0, 1, 2, 3, 4, 5, 6]**
5. If a die is thrown and a card is selected at random from a deck of 52 cards. What is the probability of getting an even number on the die and a spade card ? [Answer : 1/8]

VSA TYPE_2 MARKS QUESTIONS (SOLVED)

1. **A bag contains 5 white, 7 red and 3 black balls. If three balls are drawn one by one without replacement, find what is the probability that none is red.**

Solution : $P(nonred).P(nonred).P(nonred) = \frac{8}{15} \times \frac{7}{14} \times \frac{6}{13} = \frac{8}{65}$

2. **Find K if the following probability distribution is possible.**

X	**0**	**1**	**2**	**3**
P (X)	**K**	**K^2**	**K**	**0.04**

Solution :

We have $\sum$ P (X_i) = 1

K +k^2+k +.04=1

or $K^2+2K+.96=0$

or $K = 0.4, -2.4$

$\therefore$ $K \neq -2.4$ $\therefore$ $K = 0.4$

3. **If** $P(A)=0.4\ P(B)=p,\ P(A\cap B)=0.7$. **Find the value of p, if A and B are independent.**

Solution. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\therefore\ P(A\cap B)=0.4+p-0.7=p-0.3$

$\because$ A and B are independent

$\therefore$ $P(A\cap B)=P(A).P(B)$

is $(p-0.3)=0.4p$

$0.6p=0.3$

$p=\frac{1}{2}$

4. **If P(A) = 0.6 , P(B) = 0.2 and P($\frac{A}{B}$) $= 0.5$ find P(A'/B').**

Solution: Since P($\frac{A}{B}$) $= \frac{P(A\cap B)}{P(B)} \Rightarrow$ 0.5 = $\frac{P(A\cap B)}{0.2} \Rightarrow$ $P(A\cap B)$= 0.5 x 0.2 =0.1

$P(A'\cap B') = P(AUB)'$

$= 1 - P(A\cup B)$

$= 1 - [P(A)+P(B)-P(A\cap B)]$

$= 1 - [0.6+0.2-0.1]$

$= 1 - 0.7$

$= 0.3$

$P(A'/B') = \frac{P(A' \cap B')}{P(B')}$

$= \frac{0.3}{0.8}$

$= \frac{3}{8}$

5. **A random variable X has a probability distribution P (X) of the following form where k is some number P(X)**

$$P(X) = \begin{cases} k & if \quad X = 0 \\ 2k & if \quad X = 1 \\ 3k & if \quad X = 2 \\ 0 & 0 \quad \text{otherwise.} \end{cases}$$

Determine $P(X \leq 2)$

Solution. $\sum P(X) = 1 \Rightarrow k + 2k + 3k + 0 = 1 \Rightarrow k = 1/6$

$P(X \leq 2) = k + 2k + 3k = 6k = 1$

VSA TYPE_2 MARKS QUESTIONS (UNSOLVED)

1. Given that the events A and B are such that P(A) =1/2 ,P(A ∪ B) =3/5 and P(B)=K. Find K if they are independent. [**Answer : k=5**]

2. A random variable X has a probability distribution P(X) of the following form where k is some number

P(X)= k, if X=0
2k, if X=1
3k, if X=2
0 , otherwise

Determine i) k ii) P(X<2) [**Answer : (i)k=1/6,(ii) 1/2**]

3. Find the probability distribution of a number of a tails in the simultaneous tosses of three coins. [**Answer :**

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

4. A die marked 1,2, 3 in red and 4, 5, 6 in green is tossed . Let A be the event , 'the number is even,'and B be the event ,' the number is red'. Are A and B independent?

[**Answer : NO]**

5. In a family the husband tells a lie in 30% cases and the wife in 35% cases. Find the probability that both state the same fact. [**Answer : 0.56]**

SA TYPE_3 MARKS QUESTIONS (SOLVED)

Q.1 **In a school, there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII, what is the probability that a student chosen randomly studies in class XII, given that the chosen student is girl ?** **Solution.** P(Girl) = $P(G) = \frac{430}{1000} = \frac{43}{100}$

P (A) = Prob. of Cl XII std.

P(Girl and student of Cl XII) = $P(A \cap G) = \frac{43}{430} = \frac{1}{10}$

$$P(A/G) = \frac{P(A \cap G)}{P(G)} = \frac{\frac{1}{10}}{\frac{43}{100}} = \frac{100}{10 \times 43}$$

$= \frac{10}{43}$ **Ans.**

Q.2. 12 cards, numbered 1 to 12 are placed in a box, mixed up thoroughly and then *a* card is drawn at random from the box. If it is known that the number on the drawn card is more than 3, find the probability that it is even number.

Solution S = {1, 2, 3, 4, ...12}

A = {2, 4, 6, 8, 10, 12}

B = {4, 5, 6, 7, 8, 9, 10, 11, 12}

and $A \cap B$ = {4, 6, 8, 10, 12}

$$\therefore \quad P(A)=\frac{6}{12}=\frac{1}{2} \quad ; P(B)=\frac{9}{12}=\frac{3}{4} ; P(A \cap B)=\frac{5}{12}$$

$$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{5/12}{9/12}=5/9$$

Q.3. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of success.

Solution. : $ ={(1, 1), (2, 2), (3, 3), (4, 4), (5, 5) (6, 6)}

p (a doublet) = $\frac{1}{6}$

$$q=5/6$$

$$P(X=0)={}^4C_0\left(5/6\right)^4=\frac{625}{1296}$$

$$P(X=1)={}^4C_1\left(1/6\right)\left(5/6\right)^3=\frac{500}{1296}$$

$$P(X=2)={}^4C_2\left(1/6\right)^2\left(5/6\right)^2=\frac{150}{1296}$$

$$P(X=3)={}^4C_3\left(1/6\right)^3\left(5/6\right)^4=\frac{20}{1296}$$

$$P(X=4)={}^4C_4\left(1/6\right)^4=\frac{1}{1296}$$

$\therefore$ Required probability Distribution is :

X	0	1	2	3	4
P(X)	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$

Q.4. A letter is known to have come from either TATANAGAR or CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letter has come from

(i) Tata Nagar **(ii) Calcutta**

Solution : A : TATANAGAR (2 T'S ; 4A'S)

B : CALCUTTA (2 T'S ; 2A'S)

E : TA is visible

P(A) = P (B) = $\frac{1}{2}$

$$P(E/A) = \frac{2}{8} = \frac{1}{4} \; ; \; P(E/B) = \frac{1}{7}$$

(i) $$P(A/E) = \frac{\frac{1}{2} \times \frac{1}{4}}{\frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{7}} = \frac{1}{7}$$

(ii) $$P(B/E) = 1 - \frac{7}{11} = \frac{4}{11}$$

Q.5. **Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of number of aces.**

Solution. X = Number Aces ; X can take values 0, 1, 2

S = getting an ace.

$$P(S) = \frac{4}{52} = \frac{1}{13} \quad ; \quad P(\bar{S}) = \frac{12}{13}$$

$$P(X=0) = {}^2C_0 \frac{12}{13} \times \frac{12}{13} = \frac{144}{169}$$

$$P(X=1) = {}^2C_1 \frac{12}{13} \times \frac{1}{13} = \frac{24}{169}$$

$$P(X=2) = {}^2C_2 \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$$

∴ Probability Distributions

X	0	1	2
P(X)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Q.6. **A bag X contain 2 white and 3 red alls and a bag Y contain 4 white and 5 red balls. One ball is drawn at random from one of the bag and is found to be red . Find the probability that it was drawn from bag Y.?**

Solution.

Let E_1 : the bag X is chosen.

E_2: the bag Y is chosen.

A : the ball is red.

Using Baye's Theorem ,
$$P(E_2/A) = \frac{P(E_2)\, P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)\, P(A/E_2)}$$

Since the two bags are equally likely to be selected.

$P(E_1) = P(E_2) = ½$

Also $P(A/E_1) = 3/5$

$P(E_2) = 5/9$

$$P(E_2/A) = \frac{\frac{1}{2}\times\frac{5}{9}}{\frac{1}{2}\times\frac{3}{5}+\frac{1}{2}\times\frac{5}{9}} = \frac{25}{52}$$

Q.7.**A coin is biased so that the head is 3 time as likely to occur as tail . If the coin is tossed twice, find the probability distribution of number of tails.**

Solution.

Let p be the probability of obtaining a head when a coin is tossed once and q, that of obtaining a tail so that p = 3q and p+q=1

or 3q+q=1 or 4q=1 or q=1/4 And hence p= 3q = ¾

Let X denote the number of tails in two tosses of the coin then X can take value 0,1, 2

P (X=0) = P (HH) = p.p = p^2 = $(3/4)^2$ = 9/16

P(X=1) = P(HT,TH) =P(HT) + P(TH)

= qp + qp = 2pq =2 × ¾ × ¼ = 6/16

P(X=2) = P(two tails) = P(TT) = q^2 = $(1/4)^2$ = 1/16

Probability distribution of X is

X	0	1	2
P(X)	9/16	6/16	1/16

Q.8. The probability of A hitting a target is $\frac{3}{7}$ and that of B hitting is $\frac{1}{3}$. They both fire at the target. Find the probability that (i) at least one of them will hit the target, (ii) Only one of them will hit the target.

Solution. Let A= event that A will hits the target, B= event that B will hits the target

$$P(A)=\frac{3}{7},P(B)=\frac{1}{3}\therefore P(\overline{A})=1-\frac{3}{7}=\frac{4}{7},P(\overline{B})=1-\frac{1}{3}=\frac{2}{3}$$

$$(i)P(\text{at least one of them will hit the target})=1-P(\textit{none}\text{of them will hit the target})$$

$$=1-P(\overline{A}).P(\overline{B})=1-\frac{4}{7}\times\frac{2}{3}=\frac{13}{21}$$

$$(ii)P(\text{only one of them will hit the target})=P(A).P(\overline{B})+P(\overline{A}).P(B)=\frac{3}{7}\times\frac{2}{3}+\frac{4}{7}\times\frac{1}{3}=\frac{10}{21}$$

SA TYPE_3 MARKS QUESTIONS (UNSOLVED)

1.Given that the two numbers appearing on throwing two dice are different . Find the probability of the event ' the sum of number on the dice is 4'. [**Answer : 1/15**]

2. A box contains 16 bulbs out of which 4 bulbs are defective . 3 bulbs are drawn one by one from the box without replacement . Find the probability distribution of the number of defective bulbs drawn. [**Answer :**

X	0	1	2	3
P(X)	55/140	66/140	18/140	1/140

3. A die is thrown three times. Events A and B are defined as below:

A: 4 on the third throw. B: 6 on the first and 5 on the second throw.

Find the probability of A, given that B has already occurred. [**Answer :1/6**]

4. An insurance company insured 2000 scooter and 3000 motorcycles . The probability of an accident involving scooter is 0.01 and that of motorcycle is 0.02 . An insured vehicle met with an accident. Find the probability that the accidental vehicle was a motorcycle. [**Answer :3/4**]

5. A bag contains 5 white and 6 black balls and another bag contains 4 white and 3 black balls. A ball is drawn from the first bag and without seeing its colour is put in the second bag. Find the probability that if now a ball is drawn from the second bag it is black in colour.[**Answer :39/88**]

6. A letter is known to have come either from LONDON or CLIFTON. On the envelope just has two consecutive letters ON are visible. What is the probability that the letter has come from (i) LONDON (ii) CLIFTON ? [**Answer : (i)**12/17**(ii)** 5/17]

7. A can hit a target 4 times out of 5 times. B can hit the target 3 times out of 4 times and C can hit 2 times out of 3 times. They fire simultaneously. Find the probability that any two out of A, B and C will hit the target. [**Answer :13/30**]

8. A bag contain 1 white and 6 red balls and a second bag contains 4 white and 3 red balls. One of the bag is picked up at random and a ball is randomly drawn from it, and is found to be white in colour. Find the probability that the drawn ball was from the first bag. [**Answer :1/5**]

LA TYPE_5 MARKS QUESTIONS(SOLVED)

Q.1. A factory has three machine X,Y,Z producing 1000,2000,3000 bolts per day respectively . The machine X produced 1% defective bolts, Y produce 1.5% and Z produce 2% defective bolts . At the end of a day, a bolt is drawn at random and is found defective. What is the probability that the defective bolt is produced by the machine X ?

Solution. Let E_1: Bolt is manufactured by machine 'X', E_2: Bolt is manufactured by machine Y and E_3 : Bolt is manufactured by machine Z,

Total number of bolts manufactured by machine X,Y,Z in one day = 1,000 +2,000 +3,000 = 6,000

Therefore $P(A/E_1) = 1/6$ $P(A/E_2) = 1/3$ $P(A/E_3) = 1/2$

Let A : Bolt manufactured is defective

$P(A/E_1) = 1/100$, $P(A/E_2) = 1.5/100 = 3/200$ and $P(A/E_3) = 2/100$

Required probability = $P(E_3/A) = \dfrac{P(E_3)\,P(A/E_3)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$

$$= \frac{1/6 \times 1/100}{1/6 \times 1/100 + 1/3 \times 3/200 + \frac{1}{2} \times 2/100} = \frac{1/6}{1/6 + \frac{1}{2} + 1} = 1/10$$

Q.2. Coloured balls are distributed in three bags as shown in the following table

Bag	**Colour of the ball**		
	Red	**White**	**Black**
I	**1**	**2**	**3**

II	**2**	**4**	**1**
III	**4**	**5**	**3**

A bag is selected at random and two balls are randomly drawn from the selected bag. They happen to be black and red. What is the probability that they came from bag I.

Solution. Let E_1 : Bag I is selected ; E_2 : Bag II is selected

Let E_3 : Bag III is selected and A = A black ball and a red ball is drawn

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

$$P(A/E_1) = \frac{1\times 3}{{}^6C_2} = \frac{3}{15} = 1/5$$

$$P(A/E_2) = \frac{2\times 1}{{}^7C_2} = \frac{2}{21}$$

$$P(A/E_3) = \frac{4\times 3}{{}^{12}C_2} = \frac{4\times 3}{66} = \frac{2}{11}$$

Using Baye's Theorem

$$P(E_1/A) = \frac{\frac{1}{3}\times\frac{1}{5}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{2}{21}+\frac{1}{3}\times\frac{2}{11}} = \frac{\frac{1}{5}}{\frac{1}{5}+\frac{2}{21}+\frac{2}{11}}$$

$$= \frac{231}{551}$$

Q.3. Two cards are drawn in succession from a well shuffled deck of 52 cards, the first card being replaced , before the second is drawn . Let X denote the number of spades drawn . Find the probability distribution of X ?

Solution. The probability of getting a spade when a single card is drawn = 13/52 = ¼

or P(not getting a spade) = 1- ¼= ¾

P(X =0) = P(no spade is drawn) = P(not a spade) P (not a spade)

= ¾ × ¾= 9/16

P(X=1) = P (drawing a spade only at one draw)

= P (a spade and not a spade) + P (not a spade and a spade)

= ¼ × ¾ + ¾ × ¼ = ⅜

P(X=2) = P (drawing spades at both the draws)

= ¼ × ¼ = 1/16 Hence , the probability distribution of x is

X	0	1	2
P(X)	9/16	3/8	1/16

Q.4. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find probability distribution of the number of aces.

Solution. If X is the number of aces drawn

$$P(X=0)=\frac{{}^{4}C_{0}\times{}^{48}C_{2}}{{}^{52}C_{2}}=\frac{248\times 47}{52\times 51}=\frac{564}{663}=\frac{188}{221}$$

$$P(X=1)=\frac{{}^{4}C_{1}\times{}^{48}C_{1}}{{}^{52}C_{2}}=\frac{4\times 48\times 2}{52\times 51}=\frac{96}{663}=\frac{32}{221}$$

$$P(X=2)=\frac{{}^{4}C_{1}\times{}^{48}C_{0}}{{}^{52}C_{2}}=\frac{4\times 3\times 2}{2\times 51\times 51}=\frac{3}{663}=\frac{1}{221}$$

$\therefore$ The Probability Distribution of X is given by

X	**0**	**1**	**2**

P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

LA TYPE_5 MARKS QUESTIONS(UNSOLVED)

1. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both hearts . Find the probability of the lost card being a heart. [**Answer :11/50**]

2. In answering a question on a MCQ test a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$ .What is the probability that the student knows the answer given that he answer it correctly?[**Answer :12/13**]

3. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact present, however the test also yields as false positive result for 0.5% of the healthy person tested. If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive? [**Answer :22/133** **4**. Find the probability distribution of number of doublets in three throws of a pair of dice? [**Answer :**

X	0	1	2	3
(p)x	125/216	75/216	15/216	1/216

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Class: XII Session: 2020-21

Subject: Mathematics

Sample Question Paper (Theory)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains two **parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks
2. **Part-A** has Objective Type Questions and **Part -B** has Descriptive Type Questions
3. Both Part A and Part B have choices.

Part – A:

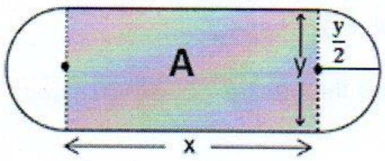
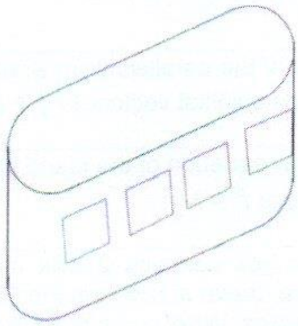
1. It consists of two sections**- I and II.**
2. Section **I** comprises of 16 very short answer type questions.
3. Section **II** contains **2** case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.

Part – B:

1. It consists of three sections**- III, IV and V.**
2. Section **III** comprises of 10 questions of **2 marks** each.
3. Section **IV** comprises of 7 questions of **3 marks** each.
4. Section **V** comprises of 3 questions of **5 marks** each.
5. Internal choice is provided in **3** questions of Section –III, **2** questions of Section-IV and **3** questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Sr. No.	Part – A	Marks
	Section I **All questions are compulsory. In case of internal choices attempt any one.**	
1	Check whether the function $f: R \rightarrow R$ defined as $f(x) = x^3$ is one-one or not. **OR**	1

	How many reflexive relations are possible in a set A whose $n(A) = 3$.	1
2	A relation R in $S = \{1,2,3\}$ is defined as $R = \{(1,1),(1,2),(2,2),(3,3)\}$. Which element(s) of relation R be removed to make R an equivalence relation?	1
3	A relation R in the set of real numbers **R** defined as $R = \{(a,b): \sqrt{a} = b\}$ is a function or not. Justify **OR** An equivalence relation R in A divides it into equivalence classes A_1, A_2, A_3. What is the value of $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$	1 1
4	If A and B are matrices of order $3 \times n$ and $m \times 5$ respectively, then find the order of matrix 5A – 3B, given that it is defined.	1
5	Find the value of A^2, where A is a 2×2 matrix whose elements are given by $a_{ij} = \begin{cases} 1 & if\ \ i \neq j \\ 0 & if\ \ i = j \end{cases}$ **OR** Given that A is a square matrix of order 3×3 and |A| = - 4. Find |adj A|	1 1
6	Let A = $[a_{ij}]$ be a square matrix of order 3×3 and |A|= -7. Find the value of $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$ where A_{ij} is the cofactor of element a_{ij}	1
7	Find $\int e^x(1 - \cot x + cosec^2 x)\, dx$ **OR** Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x\, dx$	1 1
8	Find the area bounded by $y = x^2, the\ x -$ axis and the lines $x = -1$ and $x = 1$.	1
9	How many arbitrary constants are there in the particular solution of the differential equation $\frac{dy}{dx} = -4xy^2$; y (0) = 1 **OR** For what value of n is the following a homogeneous differential equation: $\frac{dy}{dx} = \frac{x^3 - y^n}{x^2y + xy^2}$	1 1
10	Find a unit vector in the direction opposite to $-\frac{3}{4}\hat{j}$	1
11	Find the area of the triangle whose two sides are represented by the vectors $2\hat{i}\ and - 3\hat{j}$.	1

12	Find the angle between the unit vectors $\hat{a}$ *and* $\hat{b}$, given that $\mid \hat{a} + \hat{b} \mid = 1$	1
13	Find the direction cosines of the normal to YZ plane?	1
14	Find the coordinates of the point where the line $\frac{x+3}{3} = \frac{y-1}{-1} = \frac{z-5}{-5}$ cuts the XY plane.	1
15	The probabilities of A and B solving a problem independently are $\frac{1}{3}$ *and* $\frac{1}{4}$ respectively. If both of them try to solve the problem independently, what is the probability that the problem is solved?	1
16	The probability that it will rain on any particular day is 50%. Find the probability that it rains only on first 4 days of the week.	1
	Section II **Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question (17-21) and (22-26). Each question carries 1 mark**	
17	An architect designs a building for a multi-national company. The floor consists of a rectangular region with semicircular ends having a perimeter of 200m as shown below: **Design of Floor**   **Building** Based on the above information answer the following:	
	(i) If x and y represents the length and breadth of the rectangular region, then the relation between the variables is a) x + π y = 100 b) 2x + π y = 200 c) π x + y = 50 d) x + y = 100	

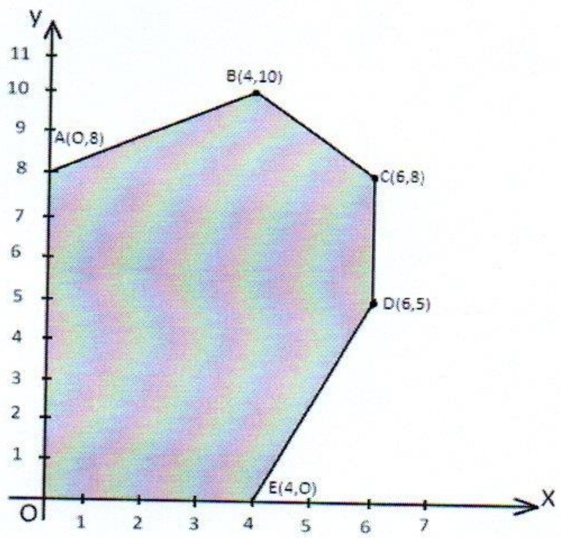
	(ii)The area of the rectangular region A expressed as a function of x is a) $\frac{2}{\pi}\left(100\,x - x^2\right)$ b) $\frac{1}{\pi}\left(100\,x - x^2\right)$ c) $\frac{x}{\pi}\left(100 - x\right)$ d) $\pi y^2 + \frac{2}{\pi}\left(100\,x - x^2\right)$	1
	(iii) The maximum value of area A is a) $\frac{\pi}{3200}m^2$ b) $\frac{3200}{\pi}m^2$ c) $\frac{5000}{\pi}m^2$ d) $\frac{1000}{\pi}m^2$	1
	(iv) The CEO of the multi-national company is interested in maximizing the area of the whole floor including the semi-circular ends. For this to happen the valve of x should be a) 0 m b) 30 m c) 50 m d) 80 m	1
	(v) The extra area generated if the area of the whole floor is maximized is : a) $\frac{3000}{\pi}m^2$ b) $\frac{5000}{\pi}m^2$ c) $\frac{7000}{\pi}m^2$ d) No change Both areas are equal	1

18	In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03  Based on the above information answer the following:	
	(i) The conditional probability that an error is committed in processing given that Sonia processed the form is : a) 0.0210 b) 0.04 c) 0.47 d) 0.06	1
	(ii)The probability that Sonia processed the form and committed an error is : a) 0.005 b) 0.006 c) 0.008 d) 0.68	1
	(iii)The total probability of committing an error in processing the form is a) 0 b) 0.047 c) 0.234	1

	d) 1	
	(iv)The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is **NOT** processed by Vinay is : a) 1 b) 30/47 c) 20/47 d) 17/47	1
	(v)Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P\ (E_i \mid \mathrm{A})$ is a) 0 b) 0.03 c) 0.06 d) 1	1
	Part – B **Section III**	
19	Express $tan^{-1}(\frac{cosx}{1-sinx})$, $\frac{-3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.	2
20	If A is a square matrix of order 3 such that $A^2 = 2A$, then find the value of |A|. **OR** If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $\mathrm{A}^2 - 5\mathrm{A} + 7\mathrm{I} = 0$. Hence find A^{-1}.	2 2
21	Find the value(s) of k so that the following function is continuous at $x = 0$	2

	$f(x) = \begin{cases} \frac{1-\cos kx}{x \sin x} & if\ x \neq 0 \\ \frac{1}{2} & if\ x = 0 \end{cases}$	
22	Find the equation of the normal to the curve y = $x + \frac{1}{x}$, $x > 0$ perpendicular to the line $3x - 4y = 7$.	2
23	Find $\int \frac{1}{cos^2x(1-tanx)^2}dx$ **OR** Evaluate $\int_0^1 x(1-x)^n\, dx$	2 2
24	Find the area of the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$.	2
25	Solve the following differential equation: $\frac{dy}{dx} = x^3\ cosec\ y, given\ that\ y(0) = 0.$	2
26	Find the area of the parallelogram whose one side and a diagonal are represented by coinitial vectors $\hat{\imath}$ - $\hat{\jmath}$ + $\hat{k}$ and 4$\hat{\imath}$ + 5$\hat{k}$ respectively	2
27	Find the vector equation of the plane that passes through the point (1,0,0) and contains the line $\vec{r}$ = λ $\hat{\jmath}$.	2
28	A refrigerator box contains 2 milk chocolates and 4 dark chocolates. Two chocolates are drawn at random. Find the probability distribution of the number of milk chocolates. What is the most likely outcome? **OR** Given that E and F are events such that P(E) = 0.8, P(F) = 0.7, P (E∩F) = 0.6. Find P ($\bar{E}$ | $\bar{F}$)	2 2
	Section IV **All questions are compulsory. In case of internal choices attempt any one.**	
29	Check whether the relation R in the set Z of integers defined as R = $\{(a,b) :\ a+b$ is "divisible by 2"} is reflexive, symmetric or transitive. Write the equivalence class containing 0 i.e. $[0]$.	3
30	If y = $e^{x\, sin^2 x}$ + $(\sin x)^x$, find $\frac{dy}{dx}$.	3
31	Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 2$ is not differentiable at $x = 1$	3

	OR If $x = a\ sec\,\theta\,, y = b\ tan\theta\ find\ \frac{d^2y}{dx^2}\ at\ x = \frac{\pi}{6}$	3
32	Find the intervals in which the function f given by $f(x) = \tan x - 4x, \quad x \in \left(0, \frac{\pi}{2}\right)$ is a) strictly increasing b) strictly decreasing	3
33	Find $\int \frac{x^2+1}{(x^2+2)(x^2+3)}\,dx.$	3
34	Find the area of the region bounded by the curves $x^2 + y^2 = 4, y = \sqrt{3}x\ and\ x - axis\ in\ the\ first\ quadrant$ **OR** Find the area of the ellipse $x^2 + 9\,y^2 = 36$ using integration	3 3
35	Find the general solution of the following differential equation: $x\,dy - (y + 2x^2)dx = 0$	3
	Section V **All questions are compulsory. In case of internal choices attempt any one.**	
36	If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}. Hence Solve the system of equations; $x - 2y = 10$ $2x - y - z = 8$ $-2y + z = 7$ **OR** Evaluate the product AB, where $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\ and\ B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ *Hence solve the system of linear equations* $x - y = 3$	5 5

	$2x + 3y + 4z = 17$ $y + 2z = 7$	
37	Find the shortest distance between the lines $\vec{r} = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ $and\ \vec{r} = 5\hat{\imath} - 2\hat{\jmath} + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$ If the lines intersect find their point of intersection **OR** Find the foot of the perpendicular drawn from the point (-1, 3, -6) to the plane $2x + y - 2z + 5 = 0$. Also find the equation and length of the perpendicular.	5 5
38	Solve the following linear programming problem (L.P.P) graphically. Maximize $Z = x + 2y$ subject to constraints ; $x + 2y \geq 100$ $2x - y \leq 0$ $2x + y \leq 200$ $x, y \geq 0$ **OR** The corner points of the feasible region determined by the system of linear constraints are as shown below:  Answer each of the following: (i) Let $Z = 3x - 4y$ be the objective function. Find the maximum and minimum value of Z and also the corresponding points at which the maximum and minimum value occurs.	5 5

	(ii) Let $Z = px + qy$, where $p, q > o$ be the objective function. Find the condition on p and q so that the maximum value of Z occurs at $\mathrm{B}(4,10)$ *and* $\mathrm{C}(6,8)$. Also mention the number of optimal solutions in this case.	

Class: XII Session: 2020-21

Subject: Mathematics

Marking Scheme (Theory)

Sr.No.	Objective type Question Section I	Marks
1	Let $f(x_1) = f(x_2)$ for $some\ x_1, x_2 \in R$ $\Rightarrow (x_1)^3 = (x_2)^3$ $\Rightarrow x_1 = x_2$, Hence $f(x)$ is one – one **OR** 2^6 reflexive relations	1 1
2	(1,2)	1
3	Since $\sqrt{a}$ is not defined for $a \in (-\infty, 0)$ $\therefore \sqrt{a} = b\ is\ not\ a\ function.$ **OR** $A_1 \cup A_2 \cup A_3 = A\ and\ A_1 \cap A_2 \cap A_3 = \phi$	1 1
4	3x5	1
5	$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ OR $\lvert adj\ A\rvert = (-4)^{3-1} = 16$	1
6	0	1
7	$e^x(1 - \cot x) + C$ **OR** $\because f(x)$ is an odd function $\therefore \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x\ dx = 0$	1 1
8	$A = 2\int_0^1 x^2\,dx = \frac{2}{3}[x^3]_0^1$ $= \frac{2}{3} sq\ unit$	1

9	0 **OR** 3	1 1
10	$\hat{j}$	1
11	$\frac{1}{2}\lvert 2\hat{i}\times(-3\hat{j})\rvert = \frac{1}{2}\lvert -6\hat{k}\rvert = 3\ sq\ units$	1
12	$\lvert\hat{a}+\hat{b}\rvert^2 = 1$ $\Rightarrow \hat{a}^2+\hat{b}^2+2\,\hat{a}.\hat{b} = 1$ $\Rightarrow 2\,\hat{a}.\hat{b} = 1-1-1$ $\Rightarrow \hat{a}.\hat{b} = \frac{-1}{2} \Rightarrow \lvert\hat{a}\rvert\lvert\hat{b}\rvert\cos\theta = \frac{-1}{2} \Rightarrow \theta = \pi - \frac{\pi}{3}$ $\Rightarrow \theta = \frac{2\pi}{3}$	1
13	1,0,0	1
14	(0,0,0)	1
15	$1-\frac{2}{3}\times\frac{3}{4}=\frac{1}{2}$	1
16	$\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3=\left(\frac{1}{2}\right)^7$	1
	Section II	
17(i)	(b)	1
17(ii)	(a)	1
17(iii)	(c)	1
17(iv)	(a)	1
17(v)	(d)	1
18(i)	(b)	1
18(ii)	(c)	1
18(iii)	(b)	1
18(iv)	(d)	1
18(v)	(d)	1
	Section III	
19	$tan^{-1}\left(\frac{cosx}{1-sinx}\right) = tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right]$ $tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\ sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$	$\frac{1}{2}$

	$tan^{-1}\left[cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right] = tan^{-1}\left[\tan\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]$ $tan^{-1}\left[tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right] = \frac{\pi}{4}+\frac{x}{2}$	1 $\frac{1}{2}$
20	$A^2 = 2A$ $\Rightarrow \|AA\| = \|2A\|$ $\Rightarrow \|A\|\|A\| = 8\|A\|$ $(\because \|AB\| = \|A\|\,\|B\|\ and\ \|2A\| = 2^3\|A\|)$ $\Rightarrow \|A\|\,(\|A\|-8) = 0$ $\Rightarrow \|A\| = 0$ or 8 **OR** $A^2 = \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix} = \begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$ $5A = \begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}, 7I = \begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$ $\Rightarrow A^2 - 5A + 7I = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} = 0$ $\Rightarrow A^{-1}(A^2 - 5A + 7I) = A^{-1}0$ $\Rightarrow A - 5I + 7A^{-1} = 0$ $\Rightarrow 7A^{-1} = 5I - A$ $\Rightarrow A^{-1} = \frac{1}{7}\left(\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix} - \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\right)$ $\Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}$	 $\frac{1}{2}$ 1 $\frac{1}{2}$ 1 1
21	$\underset{x\to 0}{Lt}\frac{1-\cos kx}{x\sin x} = \underset{x\to 0}{Lt}\frac{2\,sin^2\left(\frac{kx}{2}\right)}{x\sin x}$ $= \underset{x\to 0}{Lt}\frac{\frac{2\,sin^2\left(\frac{kx}{2}\right)}{x^2}}{\frac{x\sin x}{x^2}}$ $= \frac{\underset{x\to 0}{Lt}\frac{2\,sin^2\left(\frac{kx}{2}\right)}{\left(\frac{kx}{2}\right)^2}\times\left(\frac{k}{2}\right)^2}{\underset{x\to 0}{Lt}\frac{\sin x}{x}} = \frac{2\times 1\times\frac{k^2}{4}}{1}$	$1\frac{1}{2}$

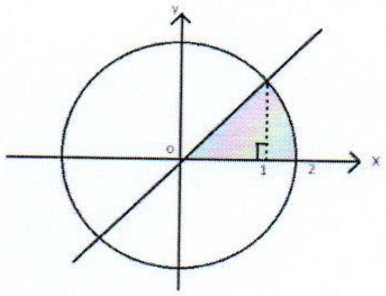
	$\because f(x)$ *is continuous at* $x = 0$ $\therefore \underset{x \to 0}{Lt} f(x) = f(0)$ $\Rightarrow \frac{k^2}{2} = \frac{1}{2} \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$	$\frac{1}{2}$
22	$y = x + \frac{1}{x} \Rightarrow \frac{dy}{dx} = 1 - \frac{1}{x^2}$ $\because$*normal is perpendicular to* $3x - 4y = 7$, $\therefore$ tangent is parallel to it $1 - \frac{1}{x^2} = \frac{3}{4} \Rightarrow x^2 = 4 \qquad \Rightarrow x = 2 \ (\because x > 0)$ *when* $x = 2$, $y = 2 + \frac{1}{2} = \frac{5}{2}$ $\therefore$*Equation of Normal* : $y - \frac{5}{2} = -\frac{4}{3}(x - 2) \Rightarrow \qquad 8x + 6y = 31$	 1 1
23	$I = \int \frac{1}{cos^2 x\,(1 - \tan x)^2} dx$ Put, $\quad 1 - \tan x = y$ So that, $-sec^2 x\, dx = dy$ $= \int \frac{-1\, dy}{y^2} = -\int y^{-2} dy$ $= +\frac{1}{y} + c = \frac{1}{1 - \tan x} + c$ **OR** $I = \int_0^1 x\,(1 - x)^n dx$ $I = \int_0^1 (1 - x)[1 - (1 - x)]^n dx$ $I = \int_0^1 (1 - x)\, x^n\, dx = \int_0^1 (x^n - x^{n+1}) dx$ $I = \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$ $I = \left[\left(\frac{1}{n+1} - \frac{1}{n+2}\right) - 0\right] = \frac{1}{(n+1)\,(n+2)}$	 1 1 $\frac{1}{2}$ 1 $\frac{1}{2}$
24	$Area = 2 \int_0^2 \sqrt{8x}\, dx$ $= 2 \times 2\sqrt{2} \int_0^2 x^{\frac{1}{2}} dx$	1

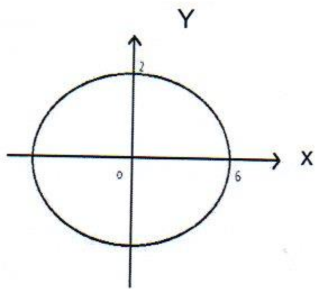
	$= 4\sqrt{2}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^2$ $= \frac{8}{3}\sqrt{2}\left[2^{\frac{3}{2}} - 0\right] = \frac{8\sqrt{2}}{3} \times 2\sqrt{2}$ $= \frac{32}{3}\ sq\ units$	$\frac{1}{2}$ $\frac{1}{2}$
25	$\frac{dy}{dx} = x^3 cosec\, y \quad ; \quad y(0) = 0$ $\int \frac{dy}{cosec\, y} = \int x^3 dx$ $\int \sin y\, dy = \int x^3 dx$ $-\cos y = \frac{x^4}{4} + c$ $-1 = c \quad (\because y = 0, when\ x = 0)$ $\cos y = 1 - \frac{x^4}{4}$	$\frac{1}{2}$ 1 $\frac{1}{2}$
26	Let $\overrightarrow{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$ $\overrightarrow{d} = 4\,\hat{\imath} + 5\hat{k}$ $\because \overrightarrow{a} + \overrightarrow{b} = \overrightarrow{d} \therefore \overrightarrow{b} = \overrightarrow{d} - \overrightarrow{a} \qquad = 3\,\hat{\imath} + \hat{\jmath} + 4\,\hat{k}$ $\overrightarrow{a} \times \overrightarrow{b} = \begin{vmatrix} \hat{\imath} - & \hat{\jmath} + & \hat{k} \\ 1 & -1 & 1 \\ 3 & 1 & 4 \end{vmatrix} = -5\hat{\imath} - 1\hat{\jmath} + 4\,\hat{k}$ Area of parallelogram = $\lvert\overrightarrow{a} \times \overrightarrow{b}\rvert = \sqrt{25+1+16} = \sqrt{42}\ sq\ units$	$\frac{1}{2}$ 1 $\frac{1}{2}$
27	Let the normal vector to the plane be $\overrightarrow{n}$ Equation of the plane passing through (1,0,0), i.e., $\hat{\imath}$ is $(\overrightarrow{r} - \hat{\imath}) \cdot \overrightarrow{n} = 0$ ………….(1) $\because$plane (1) contains the line $\overrightarrow{r} = \overrightarrow{o} + \lambda\,\hat{\jmath}$ $\therefore \hat{\imath} \cdot \overrightarrow{n} = 0$ and $\hat{\jmath} \cdot \overrightarrow{n} = 0 \quad \Rightarrow \overrightarrow{n} = \hat{k}$ Hence equation of the plane is $(\overrightarrow{r} - \hat{\imath}) \cdot \hat{k} = 0$ i.e., $\overrightarrow{r} \cdot \hat{k} = 0$	1 1
28	Let x denote the number of milk chocolates drawn <table><tr><td>X</td><td>P(x)</td></tr></table>	

	<table><tr><td>0</td><td>$\frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$</td></tr><tr><td>1</td><td>$\left(\frac{2}{6} \times \frac{4}{5}\right) \times 2 = \frac{16}{30}$</td></tr><tr><td>2</td><td>$\frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$</td></tr></table>	$1\frac{1}{2}$
	Most likely outcome is getting one chocolate of each type	$\frac{1}{2}$
	OR	
	P $(\bar{E} \mid \bar{F})$ = P $\frac{(\bar{E} \cap \bar{F})}{P(\bar{F})} = \frac{\overline{(E \cup F)}}{P(\bar{F})} = \frac{1-P(E \cup F)}{1-P(F)}$-----------(1)	1
	Now $P(E \cup F)$ = P (E) + P (F) - P (E ∩ F) = 0.8+0.7-0.6=0.9	$\frac{1}{2}$
	Substituting value of $P(E \cup F)$ in (1) P $(\bar{E} \mid \bar{F}) = \frac{1-0.9}{1-0.7} = \frac{0.1}{0.3} = \frac{1}{3}$	$\frac{1}{2}$
	Section IV	
29	(i) Reflexive : Since, a+a=2a which is even ∴ (a,a) $\in R\ \forall a \in$ Z Hence R is reflexive	$\frac{1}{2}$
	(ii) Symmetric: If (a,b) ∈R, then a+b = 2λ ⇒ b+a = 2 λ ⇒ (b,a) ∈R, Hence R is symmetric	1
	(iii) Transitive: If (a,b) ∈R and (b,c,) ∈R then a+b = 2 λ---(1) and b+c =2 μ ---- (2) Adding (1) and (2) we get a+2b+c=2(λ + μ) ⇒ a+c=2 (λ + $\mu - b$) ⇒ a+c=2k ,where $\lambda + \mu - b = k$ ⇒ (a,c) ∈R Hence R is transitive	1
	[0] = {...-4, -2, 0, 2, 4...}	$\frac{1}{2}$
30	Let u = $e^{x \sin^2 x}$and v = $(\sin x)^x$.	$\frac{1}{2}$

	so that y = u + v $\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$----(1) Now, u = $e^{x\sin^2 x}$, Differentiating both sides w.r.t. x, we get $\Rightarrow \frac{du}{dx} = e^{x\sin^2 x}\,[x(sin2x) + sin^2x]$ ----- (2) Also , v = $(\sin x)^x$ $\Rightarrow$ log v = x log $(\sin x)$ Differentiating both sides w.r.t. x, we get $\frac{1}{v}\frac{dv}{dx} = x\cot x + \log\,(sinx)$ $\frac{dv}{dx} = (\sin x)^x\ \ [x\,cotx + \log(sinx)]$ ------ (3) Substituting from $-(2), -(3)$ in $-$ (1) we get $\frac{dy}{dx} = e^{x\sin^2 x}[xsin2x + sin^2x] + (\sin x)^x\ \ [x\,cotx + \log(sinx)]$	1 1 $\frac{1}{2}$
31	RHD = $\underset{h\to 0}{Lt}\frac{f(1+h)-f(1)}{h} = \underset{h\to 0}{Lt}\frac{[1+h]-[1]}{h}$ = $\underset{h\to 0}{Lt}\frac{(1-1)}{h} = 0$ LHD = $\underset{h\to 0}{Lt}\frac{f(1-h)-f(1)}{-h} = \underset{h\to 0}{Lt}\frac{[1-h]-[1]}{-h} = \underset{h\to 0}{Lt}\frac{0-1}{-h}$ = $\underset{h\to 0}{Lt}\frac{1}{h} = \infty$ Since, RHD ≠LHD Therefore f(x) is not differentiable at x = 1 **OR** $y = b\tan\theta \Rightarrow \frac{dy}{d\theta} = b\,sec^2\theta\ ...(1)$ $x = a\sec\theta \Rightarrow \frac{dx}{d\theta} = a\sec\theta\tan\theta\ ...(2)$	1 1 1

	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\,sec^2\theta}{a\sec\theta\tan\theta} = \frac{b}{a}cosec\,\theta$	$1\frac{1}{2}$
	Differentiating both sides w.r.t. x, we get $\frac{d^2y}{dx^2} = \frac{-b}{a}cosec\,\theta\cot\theta \times \frac{d\theta}{dx}$ $= \frac{-b}{a}cosec\,\theta\cot\theta \times \frac{1}{a\sec\theta\tan\theta}$ [*using* (2)] $= \frac{-b}{a.a}cot^3\theta$	1
	$\left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{6}} = \frac{-b}{a}\left[\cot\frac{\pi}{6}\right]^3 = \frac{-b}{a}(\sqrt{3})^3 = -\frac{3\sqrt{3}b}{a.a}$	$\frac{1}{2}$
32	$f(x) = \tan x - 4x$ $f'(x) = sec^2x - 4$	$\frac{1}{2}$
	a) For $f(x)$ to be strictly increasing $f'(x) > 0$ $\Rightarrow\quad sec^2x - 4 > 0$ $\Rightarrow\quad sec^2x > 4$ $\Rightarrow\quad cos^2x < \frac{1}{4} \Rightarrow cos^2x < \left(\frac{1}{2}\right)^2$ $\Rightarrow\quad -\frac{1}{2} < \cos x < \frac{1}{2} \Rightarrow \frac{\pi}{3} < x < \frac{\pi}{2}$	$1\frac{1}{2}$
	b) For $f(x)$ to be strictly decreasing $f'(x) < 0$ $\Rightarrow\quad sec^2x - 4 < 0$ $\Rightarrow\quad sec^2x < 4$ $\Rightarrow\quad cos^2x > \frac{1}{4}$ $\Rightarrow\quad cos^2x > \left(\frac{1}{2}\right)^2$ $\Rightarrow\quad \cos x > \frac{1}{2}\ \left[\because x \in \left(0, \frac{\pi}{2}\right)\right]$ $\Rightarrow\quad 0 < x < \frac{\pi}{3}$	1

33	Put $x^2 = y\ to\ make\ partial\ fractions$ $\frac{x^2+1}{(x^2+2)(x^2+3)} = \frac{y+1}{(y+2)(y+3)} = \frac{A}{y+2} + \frac{B}{y+3}$ $\Rightarrow\ y+1 = A(y+3) + B\,(y+2)$..............(1) Comparing coefficients of y and constant terms on both sides of (1) we get A+B = 1 and 3A + 2B = 1 Solving, we get A = −1, B = 2 $\int \frac{x^2+1}{(x^2+2)(x^2+3)} dx = \int \frac{-1}{x^2+2} dx + 2\int \frac{1}{x^2+3} dx$ $= -\frac{1}{\sqrt{2}} tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{2}{\sqrt{3}} tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$	$\frac{1}{2}$ $\frac{1}{2}$ 1 1
34	Solving $y = \sqrt{3}x\ and x^2 + y^2 = 4$ We get $x^2 + 3x^2 = 4$ $\Rightarrow x^2 = 1\ \Rightarrow x = 1$  Required Area $= \sqrt{3}\int_0^1 x\ dx + \int_1^2 \sqrt{2^2 - x^2}\ dx$ $= \frac{\sqrt{3}}{2}[x^2]_0^1 + \left[\frac{x}{2}\sqrt{2^2-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$ $= \frac{\sqrt{3}}{2} + \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{6}\right]$ $\frac{2\pi}{3} sq\ units$ OR	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$

	Required Area $= \frac{4}{3}\int_0^6 \sqrt{6^2 - x^2}\, dx$  $= \frac{4}{3}\left[\frac{x}{2}\sqrt{6^2 - x^2} + 18\, sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$ $= \frac{4}{3}\left[18 \times \frac{\pi}{2} - 0\right] = 12\pi\ sq\ units$	$\frac{1}{2}$ $\frac{1}{2}$ 1 1
35	The given differential equation can be written as $\frac{dy}{dx} = \frac{y + 2x^2}{x} \quad \Rightarrow \quad \frac{dy}{dx} - \frac{1}{x}y = 2x$ $Here \quad P = -\frac{1}{x},\ Q = 2x$ IF $= e^{\int P dx} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$ The solutions is : $y \times \frac{1}{x} = \int \left(2x \times \frac{1}{x}\right) dx$ $\Rightarrow \frac{y}{x} = 2x + c$ $\Rightarrow y = 2x^2 + cx$	$\frac{1}{2}$ 1 1 $\frac{1}{2}$
36	$\lvert A \rvert = 1(-1-2) - 2(-2-0) = -3 + 4 = 1$ A is nonsingular, therefore A^{-1}exists $Adj\ A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ $\Rightarrow A^{-1} = \frac{1}{\lvert A \rvert}(Adj\ A) = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$	$\frac{1}{2}$ $1\frac{1}{2}$

	The given equations can be written as:	
	$\begin{bmatrix}1 & -2 & 0\\2 & -1 & -1\\0 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}10\\8\\7\end{bmatrix}$	$\frac{1}{2}$
	Which is of the form $A'X = B$	
	$\Rightarrow X = (A')^{-1}B = (A^{-1})'B$	1
	$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-3 & 2 & 2\\-2 & 1 & 1\\-4 & 2 & 3\end{bmatrix}\begin{bmatrix}10\\8\\7\end{bmatrix} = \begin{bmatrix}0\\-5\\-3\end{bmatrix}$	
	$\Rightarrow x = 0, \quad y = -5, \quad z = -3$	$1\frac{1}{2}$
	OR	
	$AB = \begin{bmatrix}1 & -1 & 0\\2 & 3 & 4\\0 & 1 & 2\end{bmatrix}\begin{bmatrix}2 & 2 & -4\\-4 & 2 & -4\\2 & -1 & 5\end{bmatrix}$	
	$= \begin{bmatrix}6 & 0 & 0\\0 & 6 & 0\\0 & 0 & 6\end{bmatrix}$	$1\frac{1}{2}$
	$\Rightarrow AB = 6I$ $\Rightarrow A\left(\frac{1}{6}B\right) = I \;\Rightarrow A^{-1} = \frac{1}{6}(B)$	1
	The given equations can be written as	
	$\begin{bmatrix}1 & -1 & 0\\2 & 3 & 4\\0 & 1 & 2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}3\\17\\7\end{bmatrix}$	
	$AX = D, \text{ where } D = \begin{bmatrix}3\\17\\7\end{bmatrix}$	
	$\Rightarrow X = A^{-1}D$	
	$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{6}\begin{bmatrix}2 & 2 & -4\\-4 & 2 & -4\\2 & -1 & 5\end{bmatrix}\begin{bmatrix}3\\17\\7\end{bmatrix} = \frac{1}{6}\begin{bmatrix}12\\-6\\24\end{bmatrix}$	1
	$\Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}2\\-1\\4\end{bmatrix}$	
	$x = 2, \quad y = -1, \quad z = 4$	$1\frac{1}{2}$
37	We have $a_1 = 3\hat{i} + 2\hat{j} - 4\hat{k}$ $\quad b_1 = \hat{i} + 2\hat{j} + 2\hat{k}$	

$a_2 = 5i - 2j$ $\qquad\qquad$ $b_2 = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$

$\overrightarrow{a_2} - \overrightarrow{a_1} = 2\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$ $\qquad$ 1

$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix} = \hat{\imath}(12-4) - \hat{\jmath}(6-6) + \hat{k}\,(2-6)$ $\qquad$ 1

$\overrightarrow{b1} \times \overrightarrow{b_2} = 8\hat{\imath} + 0\hat{\jmath} - 4\hat{k} = 8\hat{\imath} - 4\hat{k}$

$\because (\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} - \overrightarrow{a_1}) = 16 - 16 = 0$ $\qquad$ 1

$\therefore$ The lines are intersecting and the shortest distance between the lines is 0.

Now for point of intersection

$$3\hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) = 5\hat{\imath} - 2\hat{\jmath} + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$$

$$\Rightarrow 3 + \lambda = 5 + 3\mu \quad ---- \quad (1)$$

$$2 + 2\lambda = -2 + 2\mu \quad ---- \quad (2)$$

$$-4 + 2\lambda = 6\mu \quad ---- \quad (3)$$

1

Solving (1) ad (2) we get, $\mu = -2 \text{ and } \lambda = -4$

Substituting in equation of line we get

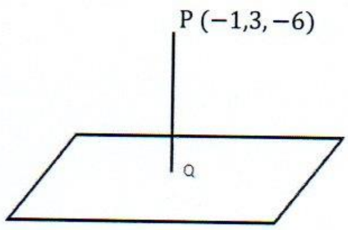
$$\vec{r} = 5i - 2j + (-2)(3\hat{\imath} + 2\hat{\jmath} - 6\hat{k}) = -\hat{\imath} - 6\hat{\jmath} - 12\hat{k}$$

Point of intersection is $(-1, -6, -12)$ $\qquad$ 1

OR

Let P be the given point and Q be the foot of the perpendicular.

Equation of PQ $\quad \frac{x+1}{2} = \frac{y-3}{1} = \frac{z+6}{-2} = \lambda$ $\qquad$ $1\frac{1}{2}$



Let coordinates of Q be $(2\lambda - 1, \lambda + 3, -2\lambda - 6)$

Since Q lies in the plane $2x + y - 2z + 5 = 0$

$$\therefore 2(2\lambda - 1) + (\lambda + 3) - 2(-2\lambda - 6) + 5 = 0$$

$$\Rightarrow 4\lambda - 2 + \lambda + 3 + 4\lambda + 12 + 5 = 0$$

$\frac{1}{2}$

$$\Rightarrow 9\lambda + 18 = 0 \qquad \Rightarrow \lambda = -2$$

$$\therefore \text{coordinates of } Q \text{ are } (-5, \quad 1, -2)$$

Length of the perpendicular = $\sqrt{(-5+1)^2 + (1-3)^2 + (-2+6)^2}$ 1

$$= 6 \text{ units}$$ 1

1

38

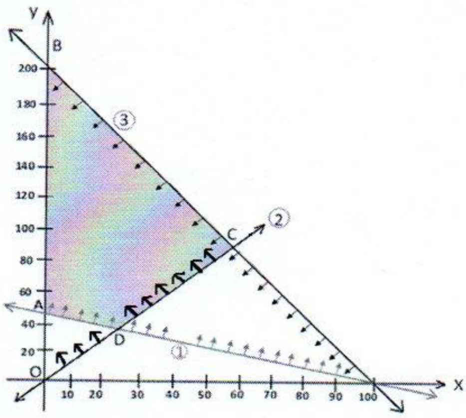
Max $Z = 3x + y$

Subject to $x + 2y \geq 100$ ----- (1)

$2x - y \leq 0$ ----- (2)

$2x + y \leq 200$ ----- (3)

$x \geq 0, \quad y \geq 0$



3

Corner Points	$Z = 3x + y$
A (0, 50)	50
B (0, 200)	200
C (50, 100)	250
D (20, 40)	100

1

$Max\ z = 250 \quad at \quad x = 50, \quad y = 100$

1

	OR	
	(i)	
	$Max \quad Z = 12 \ at \ E(4,0)$ $Min \quad Z = -32 \ at \ A(0,8)$	$1\frac{1}{2}$ 1
	(ii) Since maximum value of Z occurs at B(4,10) and C(6, 8) $\therefore 4p + 10q = 6p + 8q$ $\Rightarrow 2q = 2p$ $\Rightarrow p = q$ Number of optimal solution are infinite	2 $\frac{1}{2}$

Corner points	$Z = 3x - 4y$
O(0,0)	0
A(0,8)	-32
B(4,10)	-28
C(6,8)	-14
D(6,5)	-2
E(4,0)	12